The Welfare Economics of Foreign Aid

by

Mohammad Anisur Rahman*

INTRODUCTION

The purpose of this paper is to discuss the forces governing the demand for foreign aid by recipient countries, and the associated question of choice of domestic savings for financing economic growth, in a situation where foreign aid is available at an institutionally determined low rate of interest. The discussion rests on a highly stylized conceptual exercise in optimizing the time-pattern of investment, foreign aid, and (hence) domestic savings over a long but finite period of time, with a non-linear social preference function to be maximized subject to the attainment of a target plan terminal level of national income. The preference function to be maximized is assumed to be the sum (integral) of one-period (instantaneous) “utility” derived from aggregate consumption over the entire plan period, with marginal utility from consumption falling as consumption of any period (point of time) rises.

The exercise brings out that in the absence of offsetting political and/or psychological forces, the demand for foreign aid at the prevailing low rate(s) of interest should far exceed what recipient countries are actually obtaining currently, and foreign aid would be used not only for increasing the rate of economic growth but also for directly increasing consumption. With such excesses on the demand side, the “market” for foreign aid must be in a state of “institutional disequilibrium”, leading to “political lobbying” by recipient countries each trying to increase the allocation of a limited total amount of foreign aid in its own favour. This conclusion is contrasted with a recent theory, due to [3; 4], that emphasizes the welfare efficiency from the point of view of recipients of foreign aid of maximizing domestic savings and thereby keeping the flow of foreign aid to a minimum, a theory that we shall call the “maximum austerity”

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1 As distinct from domestic product. The distinction between GDP and GNP is being overlooked in some recent literature (e.g. in [3]) and also in Pakistan’s five year plans. This is unfortunate, and may yield misleading results.
theory. The theoretical basis of the “maximum austerity” theory, as purportedly worked out in [4], is discussed. It is concluded that the maximum austerity theory, which suggests a harmony of interests between recipients and donors of foreign aid (assuming donors, because of a limited availability of total foreign aid to distribute, want each recipient country to maximize domestic savings so as to minimize the requirement for aid), rests on the implicit assumption that there necessarily exists a “psychic disutility” from receiving foreign aid strong enough to make recipient countries unwilling to accept more aid than is minimally necessary to attain a desired rate of growth, an assumption whose validity is questionable.

II. FORMALIZATION OF THE PROBLEM AND ANALYTICAL SOLUTION

For analytical convenience the optimization problem is conceived as one of continuous time. We postulate a model with the following definitional relations:

1. \( V_t = C_t + I_t - F_t \): Gross domestic product equals consumption plus investment minus net capital inflows, defined as imports minus exports; imports do not include interest payments on outstanding external debt;

2. \( Y_t = V_t - iB_t \): Gross national income equals gross domestic product minus interest payments;

3. \( F_t = \dot{B}_t - iB_t \): Net capital inflows equal change in external indebtedness (net total borrowing) minus interest payments.

These three relations yield:

4. \( C_t = V_t - I_t + \dot{B}_t - iB_t \).

The production function is given by the familiar Harrod-Domar equation:

5. \( \dot{V}_t = bI_t \): Marginal output-capital ratio is a constant, ‘b’.

Investment has to satisfy the following:

6. \( \dot{I}_t \leq \beta I_t \): Upper limit on the rate of growth of investment is given by a constant, \( \beta \).

The problem, presented in a general form, is to maximize \( \int_0^T U(C_t) \, dt \), subject to

7. \( Y_T = Y_0 \cdot e^{ST} \): maximize the integral of utility derived from (instantaneous) consumption over a plan period covering \( T \) time units, subject

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2 The model is in some respects similar to one used by Hamada [8], but differs from the latter in a) using a finite instead of infinite plan-horizon; b) imposing a target national income for the plan terminal year; c) regarding labour as a “free” resource; and d) following the Chenery-MacEwan-Strout specification [3; 4] about absorptive capacity.
to the condition that over this period national income should grow at a given average rate, ‘g’.

Requirement (7) imposes only a terminal condition on the growth of national income. In order to allow the model maximum freedom to reveal its basic character in relation to maximizing the postulated functional, no constraint is imposed on the time path of national income within the plan period, nor any on the amount of consumption that the nation can physically absorb during any given interval of time. A time discount is not used as diminishing marginal utility itself amounts to discounting at the margin.

III. ANALYTICAL SOLUTION

It is easy to see that if ‘b’ is higher than the rate of interest ‘i’ on external borrowing, it would be efficient to invest up to its upper limit always. In other words, relation (6) would hold as a strict equality in the optimum programme throughout. Assuming \( b > i \), we can then proceed with (6.1) \( I_t = I_o e^{bt} \), \( I_o \) given historically.

Substituting (6.1) into (5) and integrating we obtain the optimum time path of gross domestic product:

\[
(8) \quad V_t = \frac{b}{\beta} I_o e^{bt} + (V_o - \frac{b}{\beta} I_o).
\]

Normalizing \( V_o \) as unity, we have

\[
(8.1) \quad V_t = \frac{b}{\beta} I_o e^{bt} + (1 - \frac{bI_o}{\beta}).
\]

Using (6.1) and (8.1) in (4), we have

\[
(4.1) \quad C_t = \lambda_1 e^{bt} + \dot{B}_t - iB_t + \lambda_2; \quad \text{where} \quad \lambda_1 = \frac{b}{\beta}(1); \quad \lambda_2 = 1 - \frac{bI_o}{\beta}.
\]

Choice of time path of consumption, \( C_t \), now reduces to choice of the time path of external borrowing, \( B_t \).

We now have a problem in the classical calculus of variations where the time path of \( B \) is to be found so as to maximize the functional \( \int_0^T U(C_t) \, dt \), with \( C_t \) a function of \( B_t \) and \( \dot{B}_t \) as given by (4.1). The initial and terminal conditions are given by

\[
(9) B_o = \text{given (outstanding indebtedness at the beginning of the plan period)}; \quad \text{and}
\]

\(^3\) The proof is left to the reader.
(10) \( B_T = \frac{1}{i} \left( V_T - Y_T \right) \), with \( V_T \) and \( Y_T \) given by (8) and (7) respectively.

The Euler equation is \( B_B \frac{d}{dt} B_B^{'} = 0 \), or \( -i \cdot U' (C_t) = U'' (C_t) \cdot \dot{C}_t \) (using (4.1), whence

\[
(11) \frac{\dot{C}_t}{C_t} = \frac{i}{\varepsilon}, \quad \text{where} \quad \varepsilon = \frac{-C_t}{U'' (C_t)} \cdot U'' (C_t)
\]

Assuming \( U'' (C_t) < 0 \), i.e., diminishing marginal utility from instantaneous consumption, \( \varepsilon \) stands for the absolute value of the point-elasticity of marginal utility. Equation (11) then says that optimum rate of growth of consumption at every point of time equals the ratio of the rate of interest on external borrowing over the absolute value of the elasticity of marginal utility from consumption at that point of time.

For simplifying the rest of the analysis, we shall assume that the elasticity of marginal utility from consumption is a constant, so that \( U(C_t) \) may be represented as

\[
(12) \quad U(C_t) = \frac{1}{1-v} C_t^{1-v}, \quad \text{a form that is already familiar in the optimum-growth literature}^4, \quad \text{with 'v' standing for the constant elasticity of marginal utility from instantaneous consumption.}
\]

The optimality condition (11) then becomes

\[
(11.1) \frac{\dot{C}_t}{C_t} = \frac{i}{v}.
\]

Combining (11.1) with (4.1) and arranging terms, we have the differential equation

\[
(13) \frac{d^2 B}{dt^2} - \frac{i(1+v)}{v} \frac{d B}{dt} + \frac{i^2}{v} B_t - \frac{(i-v \beta)}{v} \cdot \lambda_1 \cdot e^{\beta t} - \frac{i}{v} \lambda_2 = 0,
\]

with initial and terminal conditions given by (9) and (10).

The solution of this differential equation is given by

\[
(14) B_t = \frac{1}{\Delta} \left( e^{iT} \quad \infty_1 - \infty_2 e^{iT} \varepsilon e^{iT} - \frac{\lambda_1}{\beta-i} e^{\beta t} + \frac{\lambda_2}{i} \right), \quad \text{where}
\]

\[
\Delta = e^{iT} - e^{-iT};
\]

\[
\infty_1 = B_0 + \frac{\lambda_1}{\beta-i} - \frac{\lambda_2}{i};
\]

\[
\infty_2 = B_T + \frac{\lambda_1}{\beta-i} e^{\beta T} - \frac{\lambda_2}{i}.
\]

\(^4\text{See, Chakravarty [2].}\)
Equation (14) gives, under the assumptions of the problem, the optimum time path of external borrowing during the stipulated perspective plan period. Correspondingly, the optimum time path of consumption is given by

\[ C_t = C_0 e^{-v t}, \quad \text{where} \]

\[ C_0 = i \left( \frac{1}{v} - 1 \right) \frac{1}{\Delta} (\infty_2 - e^{iT} \infty_1). \]

IV. QUANTITATIVE SOLUTIONS

In this section quantitative solutions are presented for two different values of \( 'v' \), using Pakistan's figures as the initial conditions, and the Chenery-MacEwan specifications in [4] about growth of absorptive capacity. The target growth rate for GNP is arbitrarily fixed at 7 per cent, and the rate of interest on external borrowing likewise at 6 per cent. The qualitative characters of the two solutions and the lessons obtained therefrom are believed to be of fairly general validity.

The various parameters in the two exercises have the following values:

**Initial conditions (normalized)**

\[ V_o = 1; \]

\[ I_o = .15842; B_o = .05383; Y_o = 1 - iB_o = .99677 \]

**Incremental output-capital ratio**: \( b = .33; \)

**Rate of growth of investment-absorptive capacity**: \( \beta = .13; \)

**Target growth rate for GNP**: \( g = .07; \)

**Rate of interest on external borrowing**: \( i = .06; \)

**Elasticity of marginal utility from instantaneous consumption**

\( v = 2, .6. \)

The solutions for the two values of ‘\( v \)’ are given in Tables I and II respectively.

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\(^5\) The actual values are (at current prices): \( V_o (1964/65=46906 \) million rupees, using C.S.O. wholesale price deflator on an estimate of GDP at constant prices kindly supplied by Gustav Papanek (unpublished); \( I_o (1964/65=7431 \) million rupees, from Pakistan's Third Five Year Plan; \( B_o \) (outstanding at the beginning of 1964) = 2525 million rupees, from Andrews and Mohammed [1].

An error in specification of the initial structure of Pakistan is admitted because of the discrepancy between the time argument of \( B_o \) and that of \( V_o \) and \( I_o \). This is not of major importance in the exercises designed to provide qualitative insights only.

\(^6\) Frisch's 1932 study [6] of individual elasticities ranges from well below 1 to well above 6, rising as the level of consumption falls. Although the elasticity of marginal utility from consumption in the social welfare function need not follow directly from observed individual elasticities, the latter may be presumed as a guide to a plausible range for the former.
Main Features of the Quantitative Solutions

**Solution 1: \( v = 2 \)**

1) The rate of borrowing (\( \dot{B} \), the rate of change in outstanding indebtedness) is heavy from the beginning, and rises slowly. Net capital inflows (\( F \)) is also heavy initially and rises for the first few years. After a point it starts to decline and actually becomes negative in a last phase of the plan.

(Since initially interest payments are low, the bulk of total borrowing consists of net capital inflows. Gradually, interest payments mount up, and a larger and larger portion of total borrowing is used up only to pay interest charges so that less and less net capital inflows are obtained. Towards the end of the plan total borrowing is not enough even to cover all interest payments, so that a reverse flow of net capital is generated.)

2) Consumption (\( C \)) is heavy from the beginning, and rises slowly (at the rate \( i/v, \) i.e., 3 per cent a year). It exceeds gross domestic product (\( V \)) for about the first 12 years, after which GDP catches up and surpasses consumption. Consumption exceeds gross national income (\( Y \)) until the last two years of the plan.

Gross *domestic* saving, defined as gross *domestic* product minus consumption (\( S^d \)), is accordingly negative in the first 12 years and positive thereafter. The rate of domestic saving reaches the high figure of around 55 per cent at the end of the plan. Gross *national* saving (\( S^n \)) defined as gross *national* product minus consumption, is negative until the last two years of the plan, when it becomes positive and rises sharply to reach a plan terminal rate of about 18 per cent.

3) Gross national income (\( Y \)) initially falls and then rises, slowly at first and accelerating later, reaching a plan terminal rate of growth of approximately 16 per cent per annum.

**Solution 2: \( v = .6 \)**

1) The rate of borrowing is low initially and rises rapidly over time. The time distribution of net capital inflows is skewed in the opposite direction, (\( i.e., \) biased in favour of later years) than in solution 1.

2) Consumption is lower initially than in solution 1, but grows at a much faster rate (10 per cent a year). It exceeds both domestic and national product from after the second year, so that both gross domestic saving and gross national saving are negative from this time on.
3) Gross national income rises throughout the period, accelerating less rapidly than in solution 1, and reaches a plan terminal growth rate of approximately 11 per cent per annum.

The cumulative total of net capital inflows, \( \int_0^T F_t \, dt \), is substantially higher in solution 2 than in solution 1.

V. LESSON FROM THE EXERCISE

The results of the exercise are wild quantitatively, and certainly do not give realistic policy guidance regarding the inflow of foreign capital. They bring out nevertheless, and dramatically so, an aspect of the demand for foreign capital from developing countries which has not so far been adequately recognized: the function of foreign aid is not merely to enable the recipient country to achieve a higher rate of growth than what it could have out of domestic savings pushed to its maximum, given, as defined in [3, p. 686], by "the government's ability to increase total saving by changes in the tax structure and by other policies"; foreign aid may also enable the recipient country to achieve a desired rate of growth with less austerity on its own part and thus to enjoy a higher level of consumption pari passu with growth at the desired rate. As long as the marginal output-capital ratio is higher than the rate of interest on external borrowing, a distribution of any amount of foreign aid between "higher growth" and higher current consumption is possible without imposing any burden on future generations. This is because, from any given situation, the increase in investment required to generate an increase in future income for the purpose of meeting the interest liabilities of an additional amount \( x \) of external borrowing, given by the amount \( \frac{1}{b} \cdot x \), is less than \( x \) itself if \( 'i' \) is less than \( 'b' \). The "surplus" to the amount of \( (1 - \frac{i}{b}) \cdot x \) thus made available can be distributed in any way the recipient country desires between current consumption and investment.

It is conceivable that, in the interest of intergenerational equity, a part of this surplus would be rationally allocated for current consumption\(^7\). Even if a conscious policy of intergenerational equity is not followed, foreign aid may be used as a substitute for domestic savings by governments unwilling to test its "ability to increase total saving by changes in the tax structure and by other policies".

\(^7\) In a broader sense consumption itself may be regarded as investment in human resources and has an impact on productivity through its effect on health and motivation. The separation of "consumption" from "investment", and identification only of the latter with a productive use of resources as is conventionally done in growth models, is somewhat artificial, and resulting "optimal" growth prescriptions must be taken with a grain of salt. The author is indebted to the editors of this Review for pointing this out.
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\[
\int_0^T F_t dt = 21.888
\]
A necessary condition for such substitution of domestic savings for foreign capital, whatever may be the motivation, is that foreign capital must be available in sufficient quantity so that attainment of a socially desired rate of growth does not require use of the whole supply of foreign capital, in addition to domestic savings pushed to its “maximum” as defined above, for investment. Whether a country, therefore, can use foreign aid directly for consumption or not, given a socially desired growth rate (in terms, say, of GNP), depends on the conditions of supply of foreign aid. The total supply of foreign aid available for all the developing countries taken together may be regarded as limited and, perhaps, actually short of total requirement of aid to help all the developing countries grow at their respective socially desired rates even if each country used a minimum of aid and maximized domestic savings. But the supply of foreign aid to any one individual country is a matter of bi- or multi-lateral negotiations, and can be increased within very broad limits. The terms of repayment, on the other hand, are determined more by institutional factors rather than by forces of supply and demand and it is possible, within broad limits again, for a country to obtain more foreign aid without a stiffening of the terms. A country may thus see the possibility foreign aid offers of “having the cake and eating it too” and, unless it has strong sentiments against “shopping” for aid, may be expected to attempt, by playing the necessary political game, to maximize the allocation of aid in its favour, and use foreign aid thus obtained for both higher growth and higher current consumption.

The intense political lobbying any observant eye can see in the “market” for foreign aid suggests that, at least for a large group of countries, political and/or psychological sentiments against foreign aid are less effective than an appreciation of the economic gains from foreign aid, at the prevailing institutionally determined easy terms. That part of foreign aid is being used, by countries which have been successful in turning the allocation of the limited total of aid in their own favour, as a substitute for domestic savings is also a hypothesis that fits the observations. Pakistan, for example, has during the last decade or so received a much greater flow of foreign aid—about double—as a proportion of GNP than India, and has shown a savings rate significantly lower than that of India. Chenery and Strout, for example, estimate [3, p.708] for the year 1962, from linear trends of 1957-62 data, net capital inflows at the rate of 4 per cent of GNP for Pakistan as against 2 per cent for India, and a rate of savings of 9 per cent for Pakistan as against 12 per cent for India; similar differences are also recorded by Mason [10, pp. 17 and 27]. Considering a more centralized political and administrative machinery in Pakistan than in India, one would expect the authorities in Pakistan to be capable of raising a higher rate of savings from its people “by taxation and by other policies”, than authorities in India
and its people. That the savings rate in the former has nevertheless been smaller than that in the latter suggests that the relatively greater success of its negotiations for foreign aid has induced authorities in the former to take it relatively easy on the domestic savings front, whereas authorities in the latter have been forced to squeeze domestic consumption closer to its limit by an inadequate flow of foreign aid.

Whether countries like Pakistan, which appear to have so far been successful in substituting foreign capital for domestic savings, can and will continue doing so depends mainly on two factors: a) the policy that donors of foreign aid will follow, and b) how effective political sentiments in such countries against a continuous dependence on foreign aid become. The policy of donors of aid, of which the United States is the chief and the most influential member, can easily be inferred from the following summing up of the objective of foreign aid by Professor Chenery: “In the most general sense, the main objective of foreign assistance, as of many other tools of foreign policy, is to produce the kind of political and economic environment in the world in which the United States can best pursue its own social goals” [5, p. 81]. Accordingly, foreign aid can be expected to be available—both for growth and for current consumption, one dare say—as long as a country follows policies, both national and international, contributing to an environment in which donor countries can best pursue their own social goals. As for recipients, whether or not a country will continue to follow such policies and thereby opt for the (economically) easier path to economic growth irrespective of the political costs that must be associated with such a path depends on the balance of power within the country between forces for and against such a path, a subject appropriately belonging to a separate discipline.

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8 It is being suggested in some quarters that the “loss of potential savings” in Pakistan is the result of the dominance of the “balance-of-payments gap” over the “saving-investment gap”. This explanation presupposes that the country has actually been trying to minimize the inflow of foreign capital (i.e., guided by a preference function where marginal disutility from foreign aid is always greater than marginal utility from consumption), and potential domestic savings have been unutilized because, due to certain structural characteristics of the economy, a) consumption imports could not have been lower than what they have been, and b) the portfolio of investment projects and choice of techniques in each project therein could not have been less import-intensive either in the public or in the private sector. A tall order indeed for ruling out the possibility that decision-making in these respects may actually have been induced by certain expectations about the availability of foreign exchange and the price associated with it, and that a drastic reduction in the availability of foreign exchange and/or increase in its price might have induced a choice of less import-intensive bundle of investment projects and choice of techniques, calling forth at the same time much greater domestic saving efforts. While inter-country comparisons always have an element of artificiality, the performance of Burma, which has an average saving rate of 17 per cent against India’s 12 per cent and Pakistan’s 9 per cent in the Chenery-Strout tables [3., pp. 706-707], with per capita incomes of the same order in the three countries, is strongly suggestive of what the latter countries could have achieved by way of domestic savings if they did seriously intend to minimize the inflow of foreign capital.
VI. COMMENT ON THE CHENERY-STROUT-MacEWAN "MAXIMUM AUSTERITY THEORY"

The central conclusion of our analysis is that unless offset by non-economic (e.g., political and psychological) considerations, the demand for foreign aid at the prevailing easy terms should far exceed any reasonable measure of minimum requirement of aid for a respectable rate of economic growth, and foreign aid would be used not only for accelerating the growth rate but also as a substitute for domestic savings. Domestic savings, in other words, would not be pushed to its "maximum". It may be interesting to compare this conclusion with a recent suggestion that it would be efficient, from the point of view of the recipient country's welfare, to push domestic savings to its maximum and thereby keep the flow of foreign aid to a minimum. Formerly presented by Chenery and Strout [3], this suggestion, which we shall refer to as the "maximum austerity" theory, rests for its validity on the analysis in the Chenery-MacEwan paper in this Review [4], which produces "optimum" growth paths where foreign aid is in fact minimized and domestic savings given by what has been estimated as its "maximum". It is all the more interesting to inquire about the underlying optimizing mechanism since this central characteristic persists in all the "optimum" growth paths in the Chenery-MacEwan study, each with unrestricted inter-temporal distribution of aid and each with a different supply condition of aid (i.e., no improvement in the supply condition of aid induces any relaxation of consumption). It will be shown below that the "maximum austerity" theory as inferred from the Chenery-MacEwan analysis and as an expression of the recipient country's interests, is the result of implicitly associating a "psychic" disutility with the flow of foreign aid in a linear preference function.

The Chenery-MacEwan model of the Pakistan economy does not have any return flow of resources from Pakistan to the rest of the world as a quid-pro-quo for foreign aid (F_t or foreign capital in the model). The latter is identified with the trade gap alone and thus leaves no room for debt-servicing. Unrealistic though it is, foreign aid in the model must therefore be available, within the postulated quantitative limits, free of charge. This is the only interpretation consistent with the relationships postulated in the model. For in the presence of

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See the reference to the Chenery-MacEwan analysis in page 701 of the Chenery-Strout paper. In the Chenery-Strout paper itself it is assumed that "aid is sufficiently limited—or expensive—to make the recipient country unwilling or unable to increase aid merely to increase consumption without also securing some rise in GNP", and that "the country tries to maximize consumption until the target growth rate is attained" [3, p. 686]. From these assumptions it does not follow necessarily that "there is no incentive to...increase aid by reducing saving" [3, p. 687], for the room for using aid to increase consumption after securing some rise in GNP remains logically open. The "maximum austerity" theory rests for its validity on the more rigorous analysis in the Chenery-MacEwan paper following the specific assumptions postulated therein; under these assumptions, as the Chenery-MacEwan exercises bring out domestic saving remains at the estimated maximum, and must remain so even if the supply of aid is varied short of making it infinite.
debt charges—which must take the form of interest and/or amortization in years subsequent to the year in which foreign debt is incurred—the flow of foreign aid must exceed the country’s trade deficit by the amount of debt services; in that event not only would the Chenery-MacEwan model have underestimated the flow of foreign aid, their so-called “optimum” growth paths that reach a position of trade-balance only and maintain this balance thereafter would amount to a continuing and ever-growing balance of payments deficit to which no sensible policy could remain indifferent.\footnote{Unless donors of aid agree to underwrite debt services for all times to come, in which case foreign aid, in effect, becomes a free transfer of resources from the recipients’ point of view.}

If foreign aid in the Chenery-MacEwan model is a free resource economically, what has been termed as the “price” (sometimes termed as “cost”) of foreign capital must have a purely non-economic interpretation. A close examination of this “price” reveals that this really is the case.

The price of foreign capital has been identified with a weight $\gamma$ associated with the discounted flow of foreign capital in the welfare function that has been maximized. The welfare function, in a condensed form, is postulated as $W = \sum_{t=0}^{\infty} D_t (C_t - \gamma F_t)$ where $D_t$ stands for a discount factor. In order to be dimensionally additive to the utility of consumption as represented by $D_t C_t$, the quantity $(-\gamma)$ must stand for a measure of (direct) utility of foreign aid; $\gamma$ must therefore represent in effect the disutility obtained from aid. The disutility must be psychological, arising out of political reasons and/or reasons of national pride; for foreign aid, whether used directly to increase current consumption or for investment enabling higher future consumption, produces utility otherwise, particularly if it is a free resource economically.

It may be noted that the weight $\gamma$ has been determined in the model endogenously, as the “shadow price” implicit in either or both of two constraints: (a) a requirement that foreign aid should be terminated after a specified period, and (b) a finite upper bound to the (discounted) cumulative total of foreign aid over the entire plan period. Unless there exists a psychic disutility from receiving foreign aid, there is no reason why Pakistan, or for that matter any recipient of foreign aid, should want to limit the inflow of (free) aid and/or to discontinue this flow after some time. Even with a psychic disutility expressed operationally as constraints (a) and (b) above, the “maximum austerity” theory does not necessarily follow except for the linear form of the welfare function as postulated in the Chenery-MacEwan study. With the linear form of the welfare function $\gamma$, the index of psychic disutility from receiving aid (in the base year), must exceed unity, the index of utility from (base-year) consumption;
for if $\gamma$ is less than unity, there would be a net gain in having any (and, therefore, infinite) amount of foreign aid for direct consumption always, so that the postulated constraints (a) and/or (b) above would not be satisfied. Thus to satisfy such constraints in a linear welfare function $\gamma$ must exceed unity. If $\gamma$ exceeds unity, however, there would be a net loss in using any amount of foreign aid for direct consumption, so that this function of foreign aid is precluded no matter what the relative levels of consumption and foreign aid might be for any given year. Thus foreign capital would never be used as a substitute for domestic savings, so that the latter would be pushed to its “maximum” given by the postulated saving limit, a policy that would be invariant with respect either to a change in the conditions of supply of foreign aid or to wherever the saving limit may happen to lie.

This result is typical of linear welfare functions. If one postulates a non-linear welfare function instead, with marginal utility from consumption rising as consumption falls and/or the marginal (psychic) disutility from receiving foreign aid falling as the flow of foreign aid falls, “optimum” domestic savings will not necessarily be pushed to its “maximum” irrespective of where this “maximum” happens to be, and irrespective of where the postulated upper limit to the total (cumulative) inflow of foreign aid happens to be.

One may interpret the constraints (a) and (b) above as restrictions imposed not by the recipient of aid but, instead, by the donors. The availability of foreign aid for some particular country may be limited enough to make it save its utmost for growth at a desired rate. Alternatively, foreign aid may be associated with enough “political strings” to make the recipient country unwilling to accept more than what is minimally required. In the former situation it is the supply of aid, not demand, that would dictate the flow of aid and domestic savings. A disequilibrium between supply and demand for aid in this case should be expected, unless the recipient country uses a welfare function exactly of the type that makes its demand for aid just equal the limited supply. This would be the case, for example, if the recipient country actually used the Chenery-MacEwan linear welfare function with $\gamma$ sufficiently high for exact equality between supply and demand of foreign aid. $\gamma$ must still be interpreted as the extent of psychic disutility the recipient country must have in order that supply of aid just equals demand for it. In this event, however, the operational meaning of $\gamma$ remains in question, for one cannot easily conceive an operational way for donors of aid to inspire a psychic disutility in the minds of recipients (and to make them use a linear welfare function !) to an extent that makes the latter’s demand for aid, a free commodity, exactly equal to its supply!

In the event that foreign aid is associated with “political strings” undesirable from the recipient country’s point of view, foreign aid, although free economi-
cally, would have a direct "psychological" disutility arising out of encroachment upon the recipient nation's sovereignty, and this may conceivably make the country unwilling to accept aid beyond a very minimum.

The upshot of all this is the following conclusion:

The "maximum austerity" theory, which emphasizes the welfare efficiency from the point of view of recipients of foreign aid of maximizing domestic savings and thereby minimizing the flow of foreign capital, has been deduced from an analysis that rests on a) the use of a "psychic" disutility, arising out of national pride and/or political reasons, from receiving foreign aid, in b) a linear welfare function under c) a finite upper bound to the total inflow of foreign aid during a postulated length of time. This deduction holds irrespective of the specific supply condition of foreign aid, so that the latter—e.g., rationing of foreign aid—is logically irrelevant to the theory.

While some countries—notably in the communist world and in the non-communist world Burma in recent years—have indeed exhibited a "psychological" dislike for foreign aid to the extent that they are actually trying to develop their economies with a minimum of external help, this is not true of the bulk of the countries (covered, for example, in the Chenery-Strout study) with which the economics of foreign aid as it is being developed in the Western literature is really concerned. In many of these countries a concern for "too much" dependence on foreign aid is certainly perceptible; but the degree of this concern and also the degree to which this concern has entered into effective social decision-making, varies from country to country so that the "maximum austerity" theory as a generalization of the welfare-optimizing behaviour of these countries is not credible. With the easy, institutionally determined terms at which foreign aid is being made available, many countries will find an appeal for maximum self-help, in their self-interests, unconvincing. Donors of aid must expect any attempt to ration the allocation of aid to be countered by political pressures of various forms for increased aid by those countries that see the economic gains from having foreign aid, and either are not too concerned about its political costs or think they can, by skillfully playing the political game, get away with substantial foreign aid without any significant encroachment on their national sovereignty. It may be suggested that this political game is very much in the scene.

As a corollary to the above, the practice of postulating the savings function of a country as depending only on the level of national income irrespective of the availability of external resources as is prevalent in the whole body of professional literature appears much too naive a presumption. Even more so is the presumption implied in the Chenery-Strout-MacEwan studies when they
define the “maximum” saving function as taking off from the historically given saving rate in the base year—that countries receiving foreign aid are historically saving their maximum! It is this presumption that has actually given credible growth paths in these studies notwithstanding the use in [4] of a linear welfare function that in principle ought to have pushed growth policy to an extreme, and the assumption in [3] that savings will always be pushed to its upper limit. To the best of the present author's knowledge, the only reference in the literature to the possibility that domestic savings may depend on the total amount of resources available to a country and not on current national income alone is found in a discussion between Haavelmo and Leontief\(^{11}\) [9, p. 1062]. The analysis in the present paper brings out that this suggestion may well contain an important insight hitherto not recognized.

It may be noted, finally, that the use of mathematical programming techniques is not sufficient to obtain meaningful “optimum” growth paths for an economy: concepts and results of mathematical programming must be examined carefully for economic and operational sense. Given a linear welfare function with a concept \(\gamma\) as postulated in the Chenery-MacEwan study, there does exist, mathematically, a value of \(\gamma\) that will make maximization of the postulated welfare function yield a “demand” for foreign capital exactly equal to its supply. Whether \(\gamma\) has actually an operational meaning and hence whether by manipulating \(\gamma\) in any meaningful sense donors of aid can actually control recipients' demand for aid is the more relevant economic question that must also be examined. The absence of a return flow of real resources from the recipient to donors of aid in the Chenery-MacEwan model makes untenable the authors' suggestion that the shadow price \(\gamma\) may actually be charged. On the other hand the introduction of a positive supply price of foreign aid in the sense of a return flow—that must take the form of interest and amortization not adequately represented by the concept \(\gamma\) —to be manipulated by donors of aid for specific objectives, would require significant alterations of the model and also of the welfare function: the first because foreign aid in any year would then equal the recipient country's trade balance plus debt services; the second because debt service charges represent a real cost to a nation in the form of claim of foreigners on the nation's flow of resources over and above whatever “shadow” costs one may like to impute to foreign aid. Such alterations would, of course, throw overboard all the calculations in the Chenery-MacEwan study.

\(^{11}\) Haavelmo: "I just wonder whether [the alternative hypothesis has been considered that] investment in the region is a fraction of regional income including what they get from abroad. I think we see the possible implications. It means, for example, that domestic savings could be negative if [capital import] is very large".
REFERENCES


Appendix

SOME EXPLANATORY NOTES

1. The utility function \( U(C_t) = \frac{1}{1-v} \cdot C_t^{1-v} \) has previously been used, explicitly or implicitly, in Tinbergen (Econometrica, April 1960), Goodwin (Economic Journal, December, 1961), Chakravarty (International Economic Review, September 1962) and, in a recent exposition of optimum growth theory, by Hicks (Capital and Growth, Oxford, 1965, Ch. XXI and Appendix E). Except for Hicks who wants (p. 333) total utility to be positive, such utility function is generally regarded merely as an index of social choice not related to any conscious metaphysical interpretation. With \( v > 1 \), utility as measured above is always negative, asymptotically approaching zero as consumption approaches infinity. In this case the point zero may be identified with Ramsey's "bliss" (Economic Journal, 1928). A metaphysical interpretation can also be given to such formulation of the utility function if one cares for it: it is that man is assumed to find himself basically at a position of disadvantage against "nature", and the very best he may reasonably strive for is barely to breakeven with the odds! It is not in the spirit of optimum growth theory however to mix up metaphysics with criteria of social choice.

2. The method of solution used for solving the optimization problem postulated in the paper is the method of classical calculus of variations. According to this method, a necessary condition for a functional, of the form \( \bar{f}(y(x)) = \int_{x_0}^{x'} F(x, y(x), y'(x))dx \) to be an extremum is given by

\[
\left( \frac{\partial F}{\partial y'} - \frac{d}{dx} \left( \frac{\partial F}{\partial y} \right) \right) = 0,
\]

known as the Euler Equation. In our particular case, \( t \) stands for \( x \), \( B_t \) and \( B^* \) for \( y(x) \) and \( y'(x) \) respectively; hence the whole expression \( U(C_t) \), with \( C_t = \lambda_1 e^{\delta t} + \dot{B}_t - iB_t + \lambda_2 \), stands for \( F(x, y(x), y'(x)) \). Thus the Euler Equation becomes

\[
\frac{\partial U}{\partial B} = \frac{d}{dt} \left( \frac{\partial U}{\partial B} \right) = 0.
\]

In order for the solution thus obtained to be a maximum rather than a minimum, a "second order condition" also needs to be satisfied. This condition can also be verified by classical methods, but recent advances in optimum control theory (notably Mangasarian, SIAM Journal on Control, Vol. IV, No. 1, 1966) show that this is nothing but some convexity condition(s) which in our case is satisfied by virtue of diminishing marginal utility.
3. The figures for $Y_t$, $V_t$, $B_t$, $iB_t$ and $C_t$ in Tables I and II, giving solutions to the problem with continuous time, should be interpreted as rates of flow per year at the respective points of time, very much analogous to the speed (miles per hour) of an object in motion at a point of time. Thus $C_t$, for example, is not “base year” consumption, but the rate of consumption at the point of time $t = 0$. While the initial conditions ($V_o$, $I_o$ and $B_o$) must be the same in both the tables, the policy variable $B_o$ and hence $F_o$ and $C_o$ need not, and in general will not, be the same for different specifications of the utility function. This merely means that in the continuous time version of the problem the effect of policy starts instantaneously with the beginning of the plan, and the distinction between “base year” and the “first year” of the plan period does not exist.