An Estimation of the Grant Element in Foreign Aid

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Introduction

The purpose of aid is to effect a real transfer of resources from the developed countries to the developing ones. Aid can be given through a number of arrangements which differ from one donor country to another. The three main variables that influence the terms of aid are the interest rate, the grace period and the repayment period. Another factor that may influence these terms is the possibility that the loan may be repayable in local instead of convertible currency; if this is so, then the grant element of the loan increases.¹ However, this possibility is ignored when calculating the grant element in this paper as it occurs infrequently in repayment agreements.

Briefly, the grant element, which is a measure of the degree of concession of a given loan, is calculated in two stages. The first deals with the calculation of the amount repayable on a loan in equal annual instalments and the second involves discounting these instalments by using an appropriate discount rate. The grant element, therefore, is the difference between the face value of the loan and the discounted sum of payments of interest and principal on the loan. Three observations about the grant element can be made from this statement. Firstly, the grant element is positive if the discounted sum of payments of interest and principal is less than the face value of the loan. Secondly, the grant element is zero when the former equals the latter. Thirdly, the grant element is negative if the discounted sum of payments of interest and principal is greater than the face value of the loan.

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¹The grant element will be overstated if the official exchange rate is less than the market rate of exchange, in other words, the local currency is overvalued.

The equations used for calculating the grant element are

$$Q (1+p)^B (1+r)^t + \frac{A}{r} [1-(1+r)^t] = 0 \quad \dots \dots (1)$$

and

$$\frac{A}{i(1+i)^B} \left[1 - \frac{1}{(1+i)^t} \right] \quad \dots \dots (2)$$

where:

- Q is the amount of the loaned principal.
- p is the rate of interest during the grace period.
- B is the grace period in years.
- r is the rate of interest on the loan.
- A is the equal annual instalments of debt servicing on the loan.
- t is the repayment period of the loan.
- i is the discount rate.

Equation (1) calculates the equal annual instalments of debt servicing on the loan and (2) the discounted value of these instalments.

The aim of this paper is to calculate the grant element in the loans received by Pakistan over a ten-year period, 1965 to 1975. The question asked is whether the grant element has declined, remained constant, or has increased over this time span. The second objective is to determine what effects each of the three variables, namely, the interest rate, the grace period, and the repayment period, have on the grant element. The third objective is to determine if these variables fluctuate irregularly or show a consistent trend for the period. The fourth objective is to prepare a table showing the grant element in each country's loans to Pakistan.

Four main conclusions of the project are cited briefly here. Firstly, the grant element is positive for all the loans received by the country in 1965—1975. Secondly, the grace period is the important variable relative to the repayment period and the interest rate. Thirdly, all these variables fluctuated for the period covered, 1965 to 1975. Finally, the Socialist countries of Eastern Europe as well as the Soviet Union, the World Bank, and Switzerland have low grant elements in their loans.

Methodology

As mentioned in the previous section aid is given on concessional terms.^a The question, then is, how concessional are these terms? In this section a methodology is outlined for calculating the degree of concession in the loans received by the country.

As noted earlier, the three variables that influence the concessional element of a loan are the rate of interest, the grace period, and the repayment

^aA capital transfer is not called "aid" or "official assistance" unless it is given on concessional terms.

period. If the values of these variables, as well as the amount of the principal loaned are known, then the value of the instalment repayable on the loan can be calculated. However, loans may be repaid in one of three ways. The first is in equal annual instalments where the repayment rate or total debt servicing remains constant over time. The second method involves repayment of the loan at an increasing rate, i.e., in each successive year the payable instalment increases. The third involves repayment of the loan at a decreasing rate over time. To simplify the analysis we assume that debt servicing remains constant over time.

A formula has been derived in the appendix for calculating the instalment repayable on a loan. It is:—

$$Q(1+p)^B (1+r)^t + \frac{A}{r} [1-(1+r)^t] = 0 \quad \dots \dots (1)$$

where:

- Q is the amount of the loaned principal.
- p is the rate of interest during the grace period
- B is the grace period in years.
- r is the rate of interest on the loan.
- A is the equal annual instalments of debt servicing on the loan.
- t is the repayment period of the loan.

In the formula above it should be noted that interest is charged during the grace period but is paid out when the grace period ends. Also, the rate of interest remains the same during the grace and repayment periods.

As mentioned above, the value of A, the equal annual instalment of debt servicing, can be obtained if the values of the following variables, the amount of the loan, the rate of interest, the grace period, and the repayment period are known. The next step involves the calculation of the present value of the debt servicing incurred on the loan. The formula for determining this is:

$$\frac{A}{i(1+i)^B} \left[1 - \frac{1}{(1+i)^t} \right] \quad \dots \dots (2)$$

where i is the discount rate.

To obtain an accurate measure of the grant element it is necessary to discount the future charges on a loan by using an appropriate discount rate. Ideally, the social discount rate should reflect the real productivity of capital in an economy. However, as this rate is difficult to calculate three rates of discount are arbitrarily assumed, namely 8, 10, and 12 percent. These rates are applied at different intervals over the ten year period 1965-1975. 8 percent is used for the initial section covering the years 1965-1968, 10 percent for the middle section 1968-1971, and 12 percent for the latter part of the period, 1971-1975. This step is taken because it is unrealistic to assume a fixed discount rate for a period when the rate of interest in international markets has been rising [4]. Although arbitrarily chosen, these rates are a reasonable

approximation of the opportunity cost involved if the country is to borrow at commercial rates in the international market.

Having calculated the present value of a loan from formula 2 it is simple to derive the grant element for the loan. This is given below:

$$\frac{Q - PV}{Q} \times 100$$

or

$$\frac{(1 - PV)}{Q} \times 100 \quad \dots \dots \dots (3)$$

where Q is the amount of the principal loaned and PV is its present value.

The next step involves the calculation of the weighted grant elements, interest rates, and grace and repayment periods. This is achieved simply by multiplying the grant element for each loan by the value of the loan, and dividing the sum of the products by the total quantity of the loans received in that particular year. In mathematical form, this can be represented as:

$$\frac{\Sigma \text{G.E.} \times Q}{\Sigma Q}$$

where G.E. is the grant element and Q is the quantity of the loan.

Similarly, weighted values can be calculated for each year for the other variables. The rationale for calculating these weighted values for each of the variables is to discover their trend, if any, over time.

The issue of tied aid or "mark-up" has been ignored in the methodology presented above. However, the effect of tied aid is quite apparent. The grant element of a loan is lowered by the extent of the mark-up. Although the grant element is reduced, development assistance still retains its concessionary character.

Ranking of Countries

An important consideration in ranking the countries is to separate the various supplier's credits from official development assistance or capital aid. The reason for making this distinction is that the former is extended on more or less commercial terms, with a low grant element, while the latter is more concessionary. Thus, if the two are combined, a distorted figure of the grant element emerges.

Data for the project were obtained from the following two sources:

- (1) The Pakistan Economic Survey 1974-1975, and
- (2) Unpublished Statistics from the Economics Affairs Division, Ministry of Finance, Government of Pakistan.

Historical Review

In this section a brief overview is given of the aid flows to Pakistan. Two types of concessional assistance can be identified, namely, commodity aid and project or capital aid. The former is provided for the purchase of commodities for current consumption, for example, of food grains. Project or capital aid is different in that it is used for the construction of factories, dams, steel mills and other similar investments.

Foreign aid to Pakistan commenced in 1950 and by 1974-1975 total disbursements of loans came to \$6.4 billion. A sum of \$1.1 billion has been repaid and a further sum of \$ 0.6 billion has been apportioned as Bangladesh's share of debt. Therefore, \$4.7 billion is the net outstanding debt of Pakistan at the end of fiscal year 1974-1975, [2].

Of the total aid given to Pakistan (inclusive of commodity and project aid) 68 percent was provided by the consortium countries such as Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Norway, Sweden, the U.K. and the U.S.A. The share of the international agencies came to 14 percent. These included organizations as the World Bank, the International Development Association, the International Finance Corporation, and the Asian Development Bank. Finally, the proportion of aid provided by the nonconsortium countries amounted to 18 percent. These include the Socialist states of Eastern Europe, the Soviet Union, China and the oil rich states of the Middle East, [2].

Grants, which do not have to be repaid, were the main feature of the foreign aid programme to Pakistan in the initial period. These formed 70 percent of the total assistance during 1950-1955 and 79 percent during the First Five Year Plan, 1955-1960, [2]. The ratio of grants (not to be confused with the grant element) to loans, which have to be repaid, declined to 15.6 percent during the Third Five Year Plan 1965-1970 (see Table 1). This decline continued over the next five years 1970 - 1975 when the ratio fell to 4.9 percent in 1974-1975, (Table 1).

Results

The grant element fluctuated over the period 1965-1975 and the other variables exhibited a similar behaviour. The weighted annual values of these variables are shown in Table 2. The relationship between the grant element and the other three variables, the rate of interest (r) and the grace and repayment periods (B and t respectively) is also brought out in Table 2. The rate of interest is negatively related to the grant element, as the latter increased the former decreases. The grace period (B) and the repayment period (t) are positively related to the grant element, as the latter increases so do B and t .

To check the sensitivity of the grant element with respect to the other variables, namely the rate of interest and the grace, and repayment periods, we have estimated their transformation elasticities. These may be defined as

Table 1

Aid Flows, 1965 — 1975 (in \$ million)

	1 Loans	2 Grants	3 2 as % of 1
1965—1970	2,280.750	356.274	15.6
1970—1971	810.779	57.113	7.0
1971—1972	65.142	18.735	2.8
1972—1973	500.995	28.944	5.7
1973—1974	1,212.538	26.251	2.1
1974—1975	1,066.440	48.468	4.5
1970—1975	3,655.894	179.511	4.9

Source: Economic Affairs Division, Government of Pakistan and [2].
 Column 3 is derived from columns 2 and 1 in the following manner:
 Grants/Loans x 100

Table 2

Weighted Annual Values of the Grant Element, the Rate of Interest, the Grace Period, and the Repayment Period*

Years	Grant Element (in percentage terms)	r	B (years)	t (Years)
1965—66	40.807	.0365	5.0877	26.4173
1966—67	42.954	.0395	5.8738	32.8248
1967—68	46.464	.0421	6.9099	38.8324
1968—69	43.230	.0453	4.8718	22.8656
1969—70	55.593	.0372	6.0040	33.4180
1970—71	53.688	.0285	10.4272	15.9927
1971—72	60.451	.0687	7.0510	38.7058
1972—73	70.902	.0359	8.1311	40.0478
1973—74	47.400	.0360	4.3840	14.1347
1974—75	62.222	.0300	5.6356	23.8874

*The discount rates used to calculate the grant element were 8, 10, and 12 percent for the years 1965-1968, 1968-1971, and 1971-1975 respectively.

d G.E.

G.E.

d Xi

Xi

where G.E. is the grant element and Xi is one of the three variables.³

These transformation elasticities may be estimated by regressing the natural logarithm (log) of the G.E. on the log of Xi.

The elasticities for the rate of interest, the grace period, and the repayment period are —0.07379, 0.32839, and 0.11375 respectively. Of the three variables, the grace period has the highest elasticity followed by the repayment period and the rate of interest.

For policy makers one can draw up a list of tables showing how the grant element varies with different discount rates, maturity periods, grace periods, and interest rates. For a listing of such tables see [1].

Ranking of Countries

As mentioned in the methodology section of this paper, three discount rates are chosen, 8, 10 and 12 percent. Initially, the ranking of countries by their grant elements is discussed when the discount rate is fixed for the period as a whole. It is notable that the smaller countries of Western Europe, such as Belgium, Denmark, and the Netherlands gave concessionary loans with the grant element varying between 70 and 80 percent if a discount rate of 12 percent is used [see Table 3]. Swedish assistance, after 1967-1968 was in the form of outright grants.

The terms of official development assistance from Canada, the IDA (International Development Agency), and the United Kingdom remained unchanged during this period. The grant element of their loans were 93 percent, 81 percent, and 91 percent respectively. However, the terms and conditions of US loans became marginally stiffer, their grant element declined from 75 percent in 1965 to 72 percent in 1974-1975. The International Bank for Reconstruction and Development, IBRD, did poorly; the degree of concession varied in the first three years of the period and then showed a steady decline towards the end of the period; the grant element having declined to 31 percent in 1974-1975 from a high of 45 percent in 1968-69.⁴

The socialist countries of Eastern Europe and the USSR are near the lower end of the ranking scale. Their average grant element varied slightly

³See [3] for further information.

⁴The reason IBRD did poorly is that this agency borrows funds on the International money market and relends it at a rate of interest marginally below the one it borrowed them at.

Table 3

The Grant Element (in percentage) by Country and Discount Rate, 1965-1975

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Consortium											
Belgium Cap. aid	8	—	43.430	34.006	47.869	59.598	66.097	—	66.097	66.097	66.097
	10	—	54.138	42.775	58.935	69.321	75.530	—	75.530	75.530	75.530
	12	—	62.471	49.793	67.287	76.426	82.071	—	82.071	82.071	82.071
„ Export Credit	8	—	10.978	8.831	8.831	—	—	—	—	6.659	3.868
	10	—	18.481	16.515	15.515	—	—	—	—	14.526	4.868
	12	—	25.039	23.231	23.231	—	—	—	—	21.403	12.538
Canada CIDA	8	86.191	86.191	86.191	86.191	86.191	86.191	—	86.191	86.191	86.191
	10	90.574	90.574	90.574	90.574	90.574	90.574	—	90.574	90.574	90.574
	12	93.364	93.364	93.364	93.364	93.364	93.364	—	93.364	93.364	93.364
„ EDC	8	18.218	—	—	—	—	15.303	—	—	—	—
	10	32.470	—	—	—	—	27.743	—	—	—	—
	12	43.735	—	—	—	—	37.934	—	—	—	—
France Export Credit	8	12.043	12.430	12.430	25.682	20.812	18.503	—	23.774	25.025	26.572
	10	19.456	19.456	19.456	33.960	30.399	33.152	—	33.267	34.696	36.417
	12	25.936	25.936	25.936	40.283	38.283	44.829	—	40.929	42.440	44.215
Germany Cap. aid	8	30.574	38.681	46.005	53.364	48.541	53.751	—	66.097	59.258	63.099
	10	42.343	50.234	58.160	64.760	58.472	65.052	—	75.530	69.626	72.989
	12	51.448	58.959	67.167	73.031	65.832	73.214	—	82.071	77.007	79.918
„ Export Credit	8	8.831	8.831	8.831	8.831	8.831	8.831	2.243	—	—	—
	10	16.515	16.515	16.515	16.515	16.515	16.515	10.482	—	—	—
	12	23.231	23.231	23.231	23.231	23.231	23.231	17.684	—	—	—

—Continued

Table-3 (Continued)

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Italy Export credit	8	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831
	10	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515
	12	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231
„ Cap. aid	8	—	—	—	—	—	16.966	—	—	—	—
	10	—	—	—	—	—	30.629	—	—	—	—
	12	—	—	—	—	—	41.700	—	—	—	—
Japan Cap. aid	8	18.815	18.815	20.723	22.596	22.596	—	—	35.988	—	43.156
	10	31.998	31.998	33.998	35.165	35.165	—	—	50.109	—	56.251
	12	42.632	42.632	43.980	45.304	45.304	—	—	60.709	—	65.911
„ Export credit	8	—	—	9.908	—	8.831	—	—	—	—	—
	10	—	—	17.501	—	16.515	—	—	—	—	—
	12	—	—	24.138	—	23.231	—	—	—	—	—
Netherlands Cap aid	8	29.266	51.099	51.099	61.988	58.222	—	60.611	60.611	60.611	60.611
	10	45.561	62.365	62.365	62.365	72.289	69.544	—	70.868	70.868	70.868
	12	57.581	70.675	70.675	70.675	79.465	77.430	—	78.086	78.086	78.086
„ Suppliers	8	9.908	9.908	—	—	8.831	—	—	—	—	—
	10	17.501	17.501	—	—	16.515	—	—	—	—	—
	12	24.138	24.138	—	—	23.231	—	—	—	—	—
„ Export credit	8	—	—	—	—	—	—	14.097	—	—	—
	10	—	—	—	—	—	—	17.105	—	—	—
	12	—	—	—	—	—	—	19.939	—	—	—

—Continued

Table-3 (Continued)

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Sweden Cap. aid	8	62.387	62.387	82.721	—	—	—	—	—	—	—
	10	72.179	72.179	88.205	—	—	—	—	—	—	—
	12	79.196	79.196	91.697	—	—	—	—	—	—	—
„ Export credit	8	6.659	—	—	—	—	—	—	—	—	—
	10	14.526	—	—	—	—	—	—	—	—	—
	12	21.402	—	—	—	—	—	—	—	—	—
UK Cap. aid	8	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620
	10	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619
	12	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781
„ Suppliers	8	—	8.831	8.831	10.978	10.978	8.931	—	—	—	—
	10	—	16.515	16.515	18.481	18.481	16.515	—	—	—	—
	12	—	23.231	23.231	25.039	25.039	23.231	—	—	—	—
US Aid	8	61.945	61.945	58.915	57.090	57.090	57.090	82.618	57.090	57.090	57.090
	10	69.701	69.701	67.588	66.061	66.061	66.061	87.885	66.061	66.061	66.061
	12	75.206	75.206	73.698	72.393	72.393	72.393	91.355	72.393	72.393	72.393
PL-480	8	—	61.945	58.915	57.090	57.090	57.090	57.090	57.090	57.090	57.090
	10	—	69.701	67.588	66.061	66.061	66.061	66.061	66.061	66.061	66.061
	12	—	75.206	73.698	72.393	72.393	72.393	72.393	72.393	72.393	72.393
Eximbank	8	18.564	8.311	—	13.622	11.821	10.750	—	—	15.172	—
	10	31.303	15.057	—	24.991	21.961	20.116	—	—	27.775	—
	12	41.424	21.078	—	34.535	30.694	28.298	—	—	38.234	—

—Continued

Table-3 (Continued)

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
US Suppliers	8	—	7.749	15.551	8.831	—	—	—	—	—	—
	10	—	15.524	22.631	16.515	—	—	—	—	—	—
	12	—	22.320	28.856	23.231	—	—	—	—	—	—
IBRD	8	19.250	19.604	14.557	15.000	9.816	—	—	—	6.620	5.774
	10	31.273	34.478	27.520	31.201	25.988	—	—	—	21.779	15.137
	12	41.039	45.930	37.529	44.698	38.576	—	—	—	33.961	31.168
IDA	8	82.721	82.721	82.721	82.721	82.721	82.721	—	82.721	82.721	82.721
	10	88.205	88.205	88.205	88.205	88.205	88.205	—	88.205	88.205	88.205
	12	91.697	91.697	91.697	91.697	91.697	91.697	—	91.697	91.697	91.697
ADB	8	—	—	—	8.050	—	4.077	—	33.804	46.184	46.184
	10	—	—	—	22.502	—	18.409	—	47.632	60.122	60.122
	12	—	—	—	34.202	—	30.010	—	58.013	69.899	69.899
Nonconsortium Denmark	8	61.164	—	—	—	—	70.069	—	—	—	80.222
	10	68.515	—	—	—	—	77.018	—	—	—	86.002
	12	74.236	—	—	—	—	82.128	—	—	—	89.899
Czechoslovakia Export credit	8	23.331	—	—	23.331	23.331	—	—	—	—	—
	10	29.792	—	—	29.792	29.792	—	—	—	—	—
	12	35.441	—	—	35.441	35.441	—	—	—	—	—
Poland Export credit	8	23.331	—	23.331	—	—	—	—	—	—	—
	10	29.792	—	29.792	—	—	—	—	—	—	—
	12	35.441	—	35.441	—	—	—	—	—	—	—

—Continued

Table-3 (Continued)

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Switzerland	8	5.564	—	—	5.564	5.564	5.564	—	—	—	—
	10	13.523	—	—	13.523	13.523	13.523	—	—	—	—
	12	20.480	—	—	20.480	20.480	20.480	—	—	—	—
USSR Capital aid	8	23.331	23.331	23.331	24.964	23.331	23.331	26.532	26.532	—	26.532
	10	29.792	29.792	29.792	31.732	29.792	29.792	33.575	33.575	—	33.575
	12	35.441	35.441	35.441	37.591	35.441	35.441	39.612	39.612	—	39.612
Yugoslavia Export credit	8	21.337	21.337	—	21.337	—	21.337	—	—	—	—
	10	27.967	27.967	—	27.967	—	27.967	—	—	—	—
	12	33.762	33.762	—	33.762	—	33.762	—	—	—	—
Turkey	8	—	—	—	17.271	—	—	—	—	—	—
	10	—	—	—	24.243	—	—	—	—	—	—
	12	—	—	—	30.338	—	—	—	—	—	—
Austria	8	—	—	—	—	10.978	—	—	—	17.861	—
	10	—	—	—	—	18.481	—	—	—	27.494	—
	12	—	—	—	—	25.039	—	—	—	35.687	—
China	8	—	—	—	—	—	93.332	—	—	—	—
	10	—	—	—	—	—	96.479	—	—	—	—
	12	—	—	—	—	—	98.114	—	—	—	—
Bulgaria Export credit	8	—	—	—	—	—	—	—	—	28.785	—
	10	—	—	—	—	—	—	—	—	36.387	—
	12	—	—	—	—	—	—	—	—	42.884	—

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Table-3 (Continued)

[illegible]

over this time span, ranging between 35 percent and 39 percent. Switzerland, with a grant element of 20 percent in her loans to Pakistan, did poorly, too.

Export credits and loans extended by the British, American and Dutch financial Institutions were made at almost commercial terms and thus the grant element was low, ranging between 24 and 12 percent.

The preceding discussion assumes that the discount rate is fixed for the whole period, 1965-1975. If a different rate is used for each of the three time periods, that is, 1965-1968, 1968-1971, and 1971-1975; and this is a more realistic approach then a different picture emerges. The grant element has shown an increase for all countries but it is important to note that the degree of increase is not the same for each country. This implies that some countries have improved their terms of granting assistance more than others. These countries are Belgium, France, Germany, Japan, the Netherlands, and the Asian Development Bank. Countries and international agencies that showed a small increase in their grant elements are the UK, USA, the International Development Agency, Canada, and the states of Eastern Europe, including the USSR.

Conclusion

In conclusion, several results of this project may be recapitulated. Firstly, the grant element experienced an increase for most countries and international agencies if different discount rates are used for different sections of the 10-years period 1965-1975; namely, 8 percent from 1965-1968, 10 percent for 1968-1971, and 12 percent for 1971-1975. Secondly, the variable B, the grace period, has the highest elasticity followed by the repayment period.

The U.K., Japan, Belgium, and Germany have improved their terms of granting assistance. UK aid from 1975-1976 is completely in grant form, Japan has lowered the rate of interest on her loans, and Belgium has raised the amortization period. German aid now is on the same terms as that of the IDA. Also, all Norwegian and Swedish assistance is in grant form. A new development in the past three years has been the increased aid inflow from nontraditional sources, that is, from the Middle Eastern Arab States, namely, Saudi Arabia, Libya, Qatar and Abu Dhabi. Total assistance from these nontraditional sources amounted to \$948.930 million upto March 1976 [2].

Appendix

Derivation of the Grant Element Formula

The formula for calculating the grant element of loans is given below. Suppose a country has received a loan which has to be repaid in 10 years. The position at the end of the first year after the grace period would be:

$$Q [(1+p)^B (1+r)] - A = X^1$$

at the end of the second year

$$Q [(1+p)^B (1+r)^2] - A(1+r) - A = X^2$$

at the end of the third year

$$Q [(1+p)^B (1+r)^3] - A(1+r)^2 - A(1+r) - A = X^3$$

and so on until the tenth year when the position would be

$$Q [(1+p)^B (1+r)^{10}] - A(1+r)^9 - A(1+r)^8 \dots - A = X^{10} = 0$$

This can be written as

$$Q [(1+p)^B (1+r)^{10}] - [A + A(1+r)^2 \dots + A(1+r)^9] = 0$$

simplyfying

$$Q [(1+p)^B (1+r)^{10}] - [1 + (1+r) + (1+r)^2 \dots + (1+r)^9] = 0$$

which can then be rewritten as

$$Q [(1+p)^B (1+r)^{10}] + \frac{A}{r} [1 - (1+r)^{10}] = 0$$

In the case of t years the formula can be expressed as

$$Q [(1+p)^B (1+r)^t] + \frac{A}{r} [1 - (1+r)^{10}] = 0$$

In this derivation:

- Q is the amount of the loaned principal.
- p is the rate of interest during the grace period.
- B is the grace period in years.
- r is the rate of interest.
- A is the equal annual instalments of debt servicing on the loan.
- t is the repayment period of the loan and
- $X^1 X^2 X^3$ is the outstanding principal plus interest less repayment in each year.

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