Trade Policy and Economic Integration in a Cournot Duopoly Model

YU-TER WANG, BIH-JANE LIU, and PAN-LONG TSAI

This paper investigates the policy and welfare implications of forming an economic region in the context of a Cournot duopoly model. Some theoretical results are obtained. First, the economic region lowers the external tariff (against non-partner countries) less than its pre-integration level when a sufficiently large subsidy on the imports from the partner is carried out. Second, economic integration reduces the non-partner country’s welfare. Third, although the region still gains from integration even under some partial trade liberalisation regimes, complete trade liberalisation within the region leads to higher regional welfare. Finally, trade liberalisation within the region improves the welfare of the world as a whole.

1. INTRODUCTION

The resurgence of regional economic integration since the late 1980s has caught the attention of many economists. Consequently, there have been many important contributions, both theoretical and empirical, that have been concerned with the impacts and perspectives of economic integration. For instance, Richardson (1993) shows that a small home country will lower the tariff against the nonpartner if a free trade agreement (FTA) causes the FTA partners’ price to fall below the nonpartner’s tariff-inclusive price. Panagariya and Findlay (1994) demonstrate that a FTA may lead to higher tariffs between blocs since the workers that previously lobbied for protection against the FTA partner countries will be used to lobby for protection against the rest of the world. Bond and Syropoulos (1996) study the effects of bloc size on market power and world welfare, and find that sufficiently large increases in the relative size of a bloc enhance its relative market power and cause the welfare of its members to rise above the free trade level. Levy (1997) uses a median-voter model, showing that while bilateral free-trade agreements may

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undermine political support for further multilateral trade liberalisation, they can never enhance political support for multilateral free trade. Utilising a version of the model adopted by Bond and Syropoulos (1996); Bond, et al. (2001) derive conditions under which the deepening of integration within a customs union, accompanied by a Kemp-Wan reduction in its external tariff, will be incentive-compatible both for the union and outside countries.

Nevertheless, in spite of this, the issue concerned with the implications of Article XXIV of the GATT has not been satisfactorily analysed, though it has been extensively studied [e.g., McMillan (1993)]. This article requires that (1) trade restrictions against nonpartner countries should not be increased after integration, and (2) that trade barriers within the region in which economic integration takes place should eventually be completely eliminated. It is quite clear that the intention behind the first part is to prevent the nonpartner countries from being harmed by the formation of economic blocs. As for the second requirement, it would be of interest to know whether internal trade liberalisation will improve the welfare for an economic region. In other words, we would like to know whether there is any economic rationale for the requirement of complete trade liberalisation within a regional economic bloc. These are the questions we are interested in and will attempt to answer in this paper.

We employ the familiar Cournot-Nash framework and compare two sub-game perfect equilibria, one for the pre-integration case and the other for the post-integration one. The pre-integration case consists of two stages. In the second stage, the exporting firms choose their optimal output levels, taking the tariffs imposed by the importing country as given. In the first stage, the government of the importing country decides the level of the tariffs, knowing how the second-stage Cournot equilibrium between the firms is affected by the first-stage policy choices. The post-integration case consists of three stages. The third stage is exactly the same as the second stage of the pre-integration case except that the tariffs are set by the economic region under consideration. In the second stage, the economic region determines the optimal external tariff (against the nonpartner countries), taking into account its impact on the firms’ behaviour and the given internal tariff. The internal tariff (against the partner countries) is decided in the first stage, again taking into account its impact on the latter two stages.

The rest of this paper is organised as follows. Section 2 sets up the basic model to investigate the pre-integration case. Section 3 first examines the relationship between the internal and external tariffs after the economic region is formed, and then goes on to study the welfare impact of the economic integration on the nonpartner countries. Section 4 analyses the incentive for the economic integration to take place and also the welfare impact of the economic integration on the whole world. Section 5 summarises our main results.
2. THE PRE-INTEGRATION MODEL

Assume that two exporting firms 1 and 2, which are located in countries A and B, respectively, produce differentiated goods, \( q_1 \) and \( q_2 \), and export them to country C.\(^1\) Following the familiar framework, let \( U(q_1, q_2, M) \) be country C’s utility:

\[
U(q_1, q_2, M) = V(q_1, q_2, M) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)
\]

where \( V(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \beta (q_1^2 + q_2^2)/2 - \delta q_1 q_2 \) denotes the utility from consuming the differentiated goods, and \( M \) is the utility from consuming the numeraire good whose price is set to one. The demand functions for the two goods can then be derived as follows:

\[
P_i = \alpha_i - \beta q_i - \delta q_j, \quad i \neq j, \quad i, j = 1, 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)
\]

where \( P_i \) is the price of good \( q_i \), and \( \alpha_i, \beta, \) and \( \delta \) are assumed to be positive, with \( \beta \geq \delta \).\(^2\)

Let \( c_i \) and \( f_i \) denote firm \( i \)’s marginal cost and fixed cost, respectively, and \( t_i \) be the import tariff imposed by country C on firm \( i \). Firm \( i \)’s profit function can then be written as

\[
\pi_i = (P_i - c_i - t_i)q_i - f_i, \quad i = 1, 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)
\]

Country C’s welfare function \( w \) consists of the consumer’s surplus \( CS \), and tariff revenue \( t_1 q_1 + t_2 q_2 \):

\[
w = CS + t_1 q_1 + t_2 q_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)
\]

where \( CS = V(q_1, q_2) - P_1 q_1 - P_2 q_2. \)

The structure of the game is as follows. In the first stage, country C chooses optimal tariffs. Once the tariffs are determined, two exporting firms will compete with each other in a Cournot-Nash fashion. The equilibrium can be solved by applying the backward induction method.

Given the tariffs, firm \( i \) will choose the optimal level of output to satisfy the following first-order condition:

\(^1\)A typical duopoly example is the European aircraft maker Airbus and its U.S. rival Boeing Co. in the production of large airliners. In addition, the duopoly model is used widely in the strategic trade literature, such as in Brander and Spencer (1984, 1985); Eaton and Grossman (1986); Dixit (1988); Collie (1991, 1994); Levy and Nolan (1992) and Bhattacharjea (1995) etc.

\(^2\)Note that \( \alpha_i \) indicates the size of market while a positive \( \delta \) can be used to measure the degree of substitution between the two differentiated goods. A higher \( \delta \) implies a higher degree of substitution. When \( \delta = \beta \), \( q_1 \) and \( q_2 \) are homogeneous goods. The assumption \( \beta \geq \delta \) also ensures that the second-order conditions and stability conditions discussed in the following sections are satisfied.
Using (2), the equations in (5) can be solved simultaneously to derive the Nash equilibrium output $q_i^*$ and price $P_i^*$:

\[
q_i^* = \frac{1}{H} [2(\alpha_j - c_i - t_i)\beta - (\alpha_j - c_j - t_j)\delta], \quad i \neq j, \quad i, j = 1, 2; \quad \ldots \quad (6a)
\]

\[
P_i^* = \alpha_i - \frac{1}{H} [2(\beta^2 - \delta^2)(\alpha_j - c_i - t_i) + \beta\delta(\alpha_j - c_j - t_j)], \quad i \neq j, \quad i, j = 1, 2, \quad (6b)
\]

where * denotes the equilibrium value under the pre-integration case, and $H \equiv 4\beta^2 - \delta^2 > 0$. From (6a) and (6b), the following comparative static results can be obtained:

\[
\frac{\partial q_i^*}{\partial t_i} = -\frac{2\beta}{H} < 0, \quad \frac{\partial q_j^*}{\partial t_i} = \frac{\delta}{H} > 0, \quad i \neq j, \quad i, j = 1, 2; \quad \ldots \quad \ldots \quad (7a)
\]

\[
0 < \frac{\partial P_i^*}{\partial t_i} = \frac{2\beta^2 - \delta^2}{H} < 1, \quad \frac{\partial P_j^*}{\partial t_i} = \frac{\beta\delta}{H} > 0, \quad i \neq j, \quad i, j = 1, 2. \quad \ldots \quad (7b)
\]

Thus, a decrease in tariff $t_i$ raises firm i’s exports while lowering those of firm j, and the price of good i decreases by a smaller extent than the change in tariff $t_i$.

By taking the exporting firms’ reactions to import tariffs into consideration, country C will choose a set of tariffs, $t_i^*$ ($i = 1, 2$), to maximise its welfare. By substituting (6a) and (6b) into Equation (4), and totally differentiating w with respect to $t_i$ and setting it to zero, we obtain:

\[
t_i^* = \frac{\beta X_i}{9\beta^2 - \delta^2}, \quad i = 1, 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)
\]

where $X_i \equiv 3\beta(\alpha_i c_i) - \delta(\alpha_j c_j) > 0, \quad i \neq j$.\(^3\) Equation (8) indicates that the optimal tariffs on imports are positive when economic integration does not occur [see also Brander and Spencer (1984)]. We will compare the pre-integration tariffs with the post-integration ones that will be derived in the following section and explore the policy and welfare implications of economic integration.

\(^3\)A positive $q_i$ ($i = 1, 2$) will ensure a positive $X_i$. For $q_i > 0, \forall i$, the following conditions must be satisfied: (1) $\alpha_i - c_i - t_i > 0, \quad i = 1, 2$; (2) $\alpha_2 - t_2 - [2\beta(\alpha_1 - t_1)\delta] + (2\beta\delta^2)/\delta < c_2 < \alpha_2 - t_2 - [\delta(\alpha_1 - t_1)2\beta] + ([\delta c_2] / 2\delta)$.\]
3. TARIFF POLICIES UNDER ECONOMIC INTEGRATION

In this section, we assume that countries C and A decide to integrate economically. Apparently, without a transfer scheme, country C may not have an incentive to lower or even eliminate a tariff on the imports from country A. However, if a transfer scheme exists so that a loser from integration can be compensated by the gainer, then an economic region may emerge. In such a case, the region’s joint welfare will be the main concern in the formation of an economic region, and the internal tariff \( t_1 \) and the external tariff \( t_2 \) will be chosen to maximise the regional welfare. The incentive for the economic integration to exist will be presented in Section 4.

Let \( W \) denote the region’s joint welfare which consists of country C’s welfare and partner firm A’s profit:

\[
W = CS + t_1 q_1 + t_2 q_2 + \pi_1. \tag{9}
\]

Assume that the internal tariff \( t_1 \) is determined prior to the external tariff \( t_2 \). The structure of the game here therefore is a three-stage sub-game perfect equilibrium. That is, \( t_1 \) will be chosen first and then followed by the determination of \( t_2 \) in the second stage. Given \( t_1 \) and \( t_2 \), firms will compete in the output market in the last stage. Again, the model can be solved backwards. Since the solutions to the last stage are the same as those in Section 2, Equations (6)-(7) are still applicable in this section.

Taking Equations (6) and (7) into account and given the internal tariff \( t_1 \), the optimal external tariff \( t_2^{**} \) which maximises the region’s joint welfare can be derived:

\[
t_2^{**} = \frac{a_2 - c_2}{3} + \frac{\delta}{3\beta} t_1, \tag{10}
\]

where the superscript ** indicates the post-integration case. From Equation (10), the effect of a change in the internal tariff on the optimal external tariff is

\[
0 < \frac{\partial t_2^{**}}{\partial t_1} = \frac{\delta}{3\beta} < 1. \tag{11}
\]

Thus, while lowering the internal tariff will lead to a lower external tariff, it is optimal for the economic region to lower the external tariff to a smaller extent than the internal tariff. In fact, the external tariff can be reduced at most by 1/3 of the reduction in the internal tariff when the two goods are homogeneous and demand is linear.

\(^4\)Kemp and Wan (1976) and Wooton (1988), for example, also discuss customs union issues from the perspective of regional welfare, assuming that some income redistribution mechanism exists.

\(^5\)It is worth noting that it does not matter whether the internal or external tariff is set first, or whether they are set simultaneously. The rationale is identical to the case where a monopolistic firm determines two variables sequentially or simultaneously. The proof can be found in Appendix A.

\(^6\)Here, the second-order condition is satisfied: \( \frac{\partial^2 W}{\partial t_2^2} = -3\beta/H < 0. \)
Moreover, by using (8) and (9) we have
\[ t_{2}^{**} - t_{1}^{*} = \frac{\delta}{3\beta}(t_{1} + t_{1}^{*}). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12) \]

Recall that the pre-integration tariffs \( t_{1}^{*} \) and \( t_{2}^{*} \) are positive. Therefore, the interrelationship between the internal trade policy and the external trade policy can be illustrated in Figure 1. This Figure indicates that the provision of a sufficiently large subsidy on the imports from the partner by the economic region is necessary for the external tariff to be lowered to a level less than its pre-integration level (i.e. \( t_{2}^{**} < t_{2}^{*} \)).

**Proposition 1.** In the Cournot duopoly model, the economic region will lower the external tariff from its pre-integration level only if a sufficiently large subsidy on the imports from the partner is granted.

Fig. 1.

Knowing the effect of a change in the internal tariff on the external tariff, we now examine its impact on the post-integration profit of the nonpartner firm. By totally differentiating \( \pi^{*} \) with respect to \( t_{1} \), we obtain
\[ \frac{d \pi^{2}}{d t_{1}} = \frac{\partial \pi^{2}}{\partial t_{1}} + \frac{\partial \pi^{2}}{\partial t_{2}} \frac{\partial t_{2}^{**}}{\partial t_{1}} > 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13) \]

The first and second terms in (13) are, respectively, the direct and indirect effects of a change in the internal tariff on the nonpartner firm’s profit. The direct effect is:
It is positive because a decrease in the internal tariff serves to switch country C’s demand for goods away from the nonpartner firm to the partner firm and thus lowers the nonpartner firm’s profit. The indirect effect (the second term in Equation (13)) is negative, as \( \frac{\partial t_2^{**}}{\partial t_1} > 0 \) and

\[
\frac{\partial \pi^2}{\partial t_2} = (P_2^{**} - c_2 - t_2^{**}) \frac{\partial q_2^{**}}{\partial t_2} + q_2^{**} \left( \frac{\partial P_2^{**}}{\partial t_2} - 1 \right) < 0.
\]  
(14b)

Using (6a) and (6b), it can be shown that the direct effect dominates the indirect effect. The sign of (13) is therefore positive. Thus we have

**Proposition 2.** Economic integration with tariff barriers against partners being lowered or eliminated will reduce the nonpartner country’s welfare.

We now turn to the first stage of the game, i.e. the determination of the internal tariff. Totally differentiating the region’s welfare with respect to \( t_1 \) yields:

\[
\frac{dW}{dt_1} = (P_1^{**} - c_1) \left( \frac{\partial q_1^{**}}{\partial t_1} + \frac{\partial q_2^{**}}{\partial t_2} \frac{\partial t_2^{**}}{\partial t_1} \right) - q_2^{**} \left[ \frac{\partial P_2^{**}}{\partial t_1} \frac{\partial t_2^{**}}{\partial t_1} + (\frac{\partial P_2^{**}}{\partial t_2} - 1) \frac{\partial t_2^{**}}{\partial t_1} \right]
\]

\[+ t_2^{**} \left( \frac{\partial q_2^{**}}{\partial t_1} + \frac{\partial q_2^{**}}{\partial t_2} \frac{\partial t_2^{**}}{\partial t_1} \right), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \]  
(15)

The three terms in Equation (15) respectively capture the effects of a change in the internal tariff on the partner firm’s profits, the consumer’s surplus, and the tariff revenue. Since the second-order condition is satisfied (i.e. \( \frac{d^2W}{dt_1^2} - (3 \beta^2 - \delta^2) / 3 \beta H < 0 \)), the optimal internal tariff can be obtained by setting (15) equal to zero:

\[
t_1^{**} = -\beta q_1^{**} < 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \]  
(16)

The optimal internal tariff is negative. In other words, if the region were to maximise its joint welfare, a subsidy on the imports from the partner country would be the first best policy. In fact, such a subsidy policy will lead the partner country to produce at the point where \( P_1^{**} = c_1 \) (see (5)) and thus reach the Pareto optimum. In a way similar to Dixit (1988), the subsidy serves to correct the distortion caused by the imperfect competition in the commodity market.7

Using (6a), (8), (10), and (16), we have:

7See also Syropoulos (1996).
Thus, subsidising the partner firm while at the same time reducing the tariff imposed on the nonpartner firm will be the optimal strategy for the economic region to adopt. Thus we have:

**Proposition 3.** In the Cournot duopoly model, the optimal policy package for the integrating region is to subsidise the imports from the partner country and to reduce tariff barriers on the imports from the nonpartner country.

### 4. THE INCENTIVE FOR ESTABLISHING THE ECONOMIC REGION

The previous sections assume that countries A and C have indeed agreed to form an economic region. However, for the two countries to have such an incentive, the joint welfare of the economic region must increase after the integration. This section attempts to demarcate the range for the internal tariff over which the region’s welfare increases after its formation. An interesting by-product of this analysis is that we are able to show that the second-best policy is free trade among the member countries when the optimal subsidy policy is not practically feasible.

Through a complicated calculation process, it can be shown that

$$W(t_1^*, t_2^*) - W(t_1^*, t_2^*) = -\frac{3\beta^2 - \delta^2}{6\beta H} (t_1^2 + Rt_1 - S), \ldots \ldots \ldots (18)$$

where

$$R = \frac{2\beta x_1}{3\beta^2 - \delta^2}, \quad S = \frac{\beta^2 (2\beta^2 + \delta^2) x_1^2}{(3\beta^2 - \delta^2) (9\beta^2 - \delta^2)^2}.$$ 

Therefore, the region’s joint welfare increases after integration (i.e. $W(t_1^*, t_2^*) - W(t_1^*, t_2^*) > 0$) if and only if the internal tariff satisfies

$$t_1 \in (-\frac{R}{2} - [(\frac{R}{2})^2 + S]^{1/2}, -\frac{R}{2} + [(\frac{R}{2})^2 + S]^{1/2}) = (t_1^U, t_1^L), \ldots \ldots (19)$$

where $t_1^U$ and $t_1^L$ are the upper and lower bounds. Under such circumstances, countries A and C will have incentives to form an economic region, provided that an appropriate compensation mechanism exists. Since $t_1^U > 0$ and $t_1^L < 0$, the region’s joint welfare can be improved when the imports from the partner country are subsidised or even only partially liberalised. Moreover, it can be shown that $t_1^{**} = -R/2$, and

$$t_1^{**} - t_1^L = t_1^U - t_1^{**} = [(R/2)^2 + S]^{1/2} > 0, \ldots \ldots \ldots (20)$$
That is, the post-integration optimal internal tariff (subsidy), \( t_1^{**} \), will fall at the midpoint of the range \((t_1^L, t_1^U)\). The results can be illustrated in Figure 2, where the welfare function \( W(t_1, t_2^{**}) \) is strictly concave with respect to \( t_1 \) since
\[
\frac{dW(t_1, t_2^{**})}{dt_1} = -\frac{3\beta^2 - \delta^2}{3\beta H}(t_1 - t_1^{**}),
\]
and thus
\[
\frac{d^2W(t_1, t_2^{**})}{dt_1^2} < 0.
\]

The regional welfare reaches the maximum at \( t_1 = t_1^{**} \), and decreases (increases) with \( t_1 \) when \( t_1 > (<) t_1^{**} \).

Fig. 2.

Although the joint welfare reaches the maximum level when the optimal subsidy \( t_1^{**} \) is granted, such a policy is not usually feasible [Bhattacharjea (1995)]. If for some reason a subsidy policy is not available, Figure 2 shows that complete elimination of the internal tariff will result in the highest level of welfare. Notice also that the partner countries could still jointly benefit from a partial liberalisation as
long as the post-integration internal tariff falls within the range \((0, t_i^U)\). Thus we conclude:

**Proposition 4.** Barring import subsidy, countries can benefit from economic integration even under a partial trade liberalisation regime, but complete abolishment of the tariff barriers among partner countries will result in the highest regional welfare.

In addition, defining the world welfare as \(W^T = W + \pi^T\) and using (2), (5)–(7) (13) and (15), we get

\[
\frac{dW^T}{dt_1} = -\frac{36\beta^3 - 23\beta^2\delta^2 + 3\delta^4}{9\beta H^2} (t_1 - t_1^T), \quad \ldots \quad \ldots \quad (22)
\]

where \(t_1^T = -\frac{36\beta^2(12\beta^2 - \delta^2)(\alpha_1 - c_1) - 2\beta\delta(8\beta^2 - \delta^2)(\alpha_2 - c_2)}{(36\beta^4 - 23\beta^2\delta^2 + 3\delta^4)}\).

Since \(\frac{d^2W^T}{dt_1^2} = -\frac{(36\beta^4 - 23\beta^2\delta^2 + 3\delta^4)}{9\beta H^2} < 0\), \(W^T\) is also a strictly concave function of \(t_1\). Moreover, it can be shown that \(t_{1*}^T < t_1^T < 0\) and \(\pi^T > 0\) so that we have the graph of Figure 2. Clearly, the world welfare increases when the tariff against partner countries is lowered. In other words, the welfare gains to the economic region are higher than the welfare losses to the nonpartner country. As a result, we arrive at the following proposition:

**Proposition 5.** The world’s net welfare is likely to increase with the formation of economic integration. If import subsidies among the partner countries are not allowed, the welfare gains are the largest when free trade prevails within the economic region.

An important lesson emerges from Proposition 4 and Proposition 5. When import subsidies are not available in an economic region, not only does free trade within the region lead to the highest regional welfare, but it also results in the highest net gains of the world welfare. Consequently, our results fully support the wisdom of the GATT’s Article XXIV, which requires trade barriers within any economic region be completely eliminated eventually.

5. CONCLUSION

The resurgence of economic integration since the late 1980s has led to a flurry of studies regarding their implications. From an economic viewpoint, there is no doubt that most attempts at economic integration have led to the adoption of policies aiming at reducing trade barriers against partner countries in the hope of achieving production efficiency, exploiting economies of scale, as well as accelerating economic growth. On the other hand, due to the concerns over the potentially negative impacts of economic integration on nonpartner countries, the GATT has endeavoured to regulate economic integration through its well-known Article XXIV.
In particular, this Article requires that (1) trade restrictions against nonpartner
countries not be raised after integration, and (2) that trade barriers within the
economic region eventually be completely eliminated. Apparently, the first
requirement is to prevent the nonpartner countries from being harmed by the
emergence of economic integration, while the second is based on the concept that
free trade in part of the world is better than no free trade at all. Besides investigating
the impact of economic integration on both the partner and nonpartner countries, it is
also interesting and important to see the implications of the requirements of Article
XXIV.

In attempting to answer the questions mentioned above, we employ in this
paper the Cournot duopoly model à la Brander and Spencer (1985) to study the
welfare impacts of the formation of an economic bloc, with special attention
being paid to the interdependence between the internal and external tariffs. Our
major results are as follows: (1) the economic region will lower the external tariff
(against nonpartner countries) from its pre-integration level when a sufficiently
large subsidy on the imports from the partner is granted; (2) the formation of an
economic bloc will reduce the nonpartner country’s welfare; (3) an economic
region can gain from economic integration even trade is partially liberalised
between partner countries though complete trade liberalisation within the region
will lead to higher regional welfare; (4) trade liberalisation within the region will
improve the welfare of the world as a whole. Above all, our results seem to
support the above-mentioned second requirement of Article XXIV of the GATT.
Admittedly, economic integration is a very complex phenomenon, mixing
economic and geopolitical considerations. Therefore, it is impossible for our
simple model to capture all the interesting aspects of this phenomenon. However,
we still hope that our analysis has shed some light in furthering our knowledge
about economic integration.

APPENDIX A

If the internal tariff $t_1$ and the external tariff $t_2$ are determined in the same
stage, the following two equations will be solved simultaneously:

$$
\frac{\partial W}{\partial t_1} = (P^*_1 - c_1) \left[ \frac{\partial q_1}{\partial t_1} - q_2^* \frac{\partial P^*_2}{\partial t_1} + q_2^* t_2 \frac{\partial q_2^*}{\partial t_1} \right] + \ldots + \ldots \quad (A.1)
$$

$$
\frac{\partial W}{\partial t_2} = (P^*_2 - c_1) \left[ \frac{\partial q_1}{\partial t_2} - q_2^* \frac{\partial P^*_1}{\partial t_2} + q_2^* t_2 \frac{\partial q_2^*}{\partial t_2} + q_2^* \right] + \ldots + \ldots \quad (A.2)
$$

and using (5) – (7) we obtain

$$
t_1^* = -\frac{\beta}{3\beta^2 - 8\gamma}[3\beta(\alpha_1 - c_2)], \quad \ldots + \ldots + \ldots + \ldots \quad (A.3)
$$
$t_2^{**} = \frac{\beta}{3\beta^2 - \delta^2} [\beta(\alpha_2 - c_2) - \delta(\alpha_1 - c_1)]. \quad \ldots \quad \ldots \quad \ldots \quad (A.4)$

Alternatively, if $t_2$ is determined in stage 1 and $t_1$ in stage 2, we will solve $\partial W/ \partial t_1 = 0$ first and get

$t_1^{**} = \frac{\delta}{\beta} t_2 - (\alpha_1 - c_1), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A.5)$

Based on (A.5), in the setting $t_1^{**}$ is a function of $t_2$ and hence

$$
\frac{dW}{dt_2} = (P_1^{**} - c_1)(\frac{\partial q_2^{**}}{\partial t_2} + \frac{\partial q_1^{**}}{\partial t_1} \frac{\partial t_1^{**}}{\partial t_2}) -
q_2^{**}(\frac{\partial P_2^{**}}{\partial t_2} + \frac{\partial P_1^{**}}{\partial t_1} \frac{\partial t_1^{**}}{\partial t_2} - 1) + t_2^{**}(\frac{\partial q_2^{**}}{\partial t_2} + \frac{\partial q_1^{**}}{\partial t_1} \frac{\partial t_1^{**}}{\partial t_2}). \quad \ldots \quad (A.6)
$$

Letting $dW/dt_2 = 0$ and using (5)–(7) and (A.5) yields (A.4). Next, (A.3) will be obtained by substituting (A.4) into (A.5). Similarly, we can also obtain (A.3) by substituting (8) into (17) and can arrive at (A.4) by substituting (A.3) into (10).

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