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Public / Private Investment Linkages: A Multivariate Cointegration Analysis

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I. INTRODUCTION

The issue of whether public investment crowds out or crowds in private investment has received considerable attention in the economic literature. Most of the empirical studies that examined the long run stable association between public and private investment have focused on examining this relationship for the developed countries with very little attention on the developing countries. The empirical results of these studies, however, are highly controversial.

The existing empirical studies in this area can be divided into three categories. The studies in the first category including Barro (1974), Kormendi (1983), and Feldstein (1982) have examined the empirical implications of the Ricardian equivalence hypothesis (REH). The empirical results of most of the studies in this category were supportive of the REH. Seater (1993) argues that good empirical studies generally provide evidence in support of the REH; however, some studies refute it owing to the lake of econometric accuracy.

The second group of studies Baily (1971), Aschauer (1985), Barro (1981), Monadjiemi (1993) and Rossiter (2002) are based on short-run Keynesian models. These models mulling over the demand-side effects of public spending conclude that public investment crowds out some though not all private investment (say substitutability hypothesis). The empirical findings of these studies, in general, are providing persuasive support for the substitutability hypothesis. The empirical findings of the third category of studies including Aschauer (1989), Erenburg (1993),

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Karras (1994), Ereburge and Woher (1995) and Monadjemi and Huh (1998), whereas, provided evidences that are robust to the complementarity hypothesis i.e., public investment crowds in private investment via improving the productivity of the private capital.

The studies by Aschauer (1985, 1988), Lynde and Richmand (1992) examine the role of public capital in explaining total factor productivity and the rate of the return on private capital in the United States' non-financial corporate sector. They found that public capital has positive marginal product and that private investment can be enhanced by increasing public investment. According to Lynde and Richmand (1992), public and private capital are complements in production, rather than substitutes. However, Aschauer says that some categories of public spending such as expenditure on research, roads and transports, water and power projects, education may enhance private sector's productivity and hence may complement private investment expenditure, whereas, government consumption expenditure on foods and health may substitute for private consumption expenditure on these items. Karras (1994) reports that substitutability and complementarity nature of private and public investment depends on the size of the government. He said, as the size of the government sector expands, the relationship between private and public investment turns into substitutability rather than complementarity.

This study attempts to explore the long-run relationship between public and private investment in Pakistan and discusses whether public and private investments are complements or substitutes. The distinguishing features of the present study from the previous research in this area are: (1) examine the link between public and private investment for a developing country rather than developed countries, (2) the use of the more robust tests of unit roots to check the dynamic properties of the time series, and (3) the empirical model is based on the marginal efficiency theory as well as neoclassical theory of investment.

It can be summed in the light of the results presented below that there is a co-movement between public and private investment. Normalised cointegration vector shows that public investment crowds in private investment. This piece of evidence is robustly in support of the or complementarity hypothesis.

The remainder of this paper is outlined as follows. Section II offers the theoretical discussion. In Section III, the econometric methodologies are discussed. This section also tells about data sources. The empirical results of the study are presented in Section IV. Section V summarises the key finding and concludes.

II. THEORETICAL DISCUSSION

The substitutability and complementarity nature of the public and private investment make macroeconomists to take keen interest in exploring the linkages between them. The substitution (crowding out) effect of public investment reduces

the ability of government to influence economic activities through public spending. The relationship between public and private investment, however, is complementary if public capital improves the productivity of private capital in production. In this case, an increase in public investment leads to an increase in private investment.

The Baily (1971) was the first study that explains the theoretical linkages between public and private investment. It examined the "crowding-out" effect of public spending and the degree of substitution and complementary relationship between public and private investment. The one recent study by Erenburg and Wohar (1995) based on accelerator, neoclassical and security valuation models, has been examined the cause-effect relationship between public and private investment in equipment.

The marginal efficiency theory of investment is the oldest theory of investment, which states that an increase in the market rate of interest has adverse effect on private investment via increasing the interest cost of financing. One the other side, as assumed by neoclassical theory of investment, the demand for capital depends on cost of capital and other factors. However, the accelerator theory of investment mentions that change in sales is an important determinant of private investment.

The Aschauer's (1989) study reported that both crowing out and crowding in effects appear by public spending, the latter effect is more vigorous and dominates the former, so the net effect of a rise in public investment spending is likely to raise private investment spending. Thus, it can be concluded that private investment is positively influenced by public spending and public investment spending on infrastructure crowds in rather than crowds out private investment.

The empirical model of this study is based on the marginal efficiency theory of investment and the modified form of neoclassical model that is similar to the accelerator cash flow model [see for detailed, Erenburg and Wohar (1995)]. The market rate of interest and change in output are included in the private investment function in order to capture the combined effects of cost of capital and other factors on demand for investment.[1]

III. ECONOMETRIC MODEL

Based on above theoretical framework the following error correction model as in Pesaran and Smith (1998) is proposed to investigate the linkage between public and private investment:

$$\Delta y_t = \sum_{i=1}^{p-1} \Gamma_{iy} \Delta z_{t-i} + \Pi_y \Delta z_{t-1} + \lambda_y \Psi_t + \alpha_{1y} t + \alpha_{0y} + \xi_t \qquad t = 1,2,3,...,T \quad \ldots \quad (1)$$

[1]Here, followed by Erenburg and Wohar (1995), a proxy for change in output is included to take into account the influence of other factors.

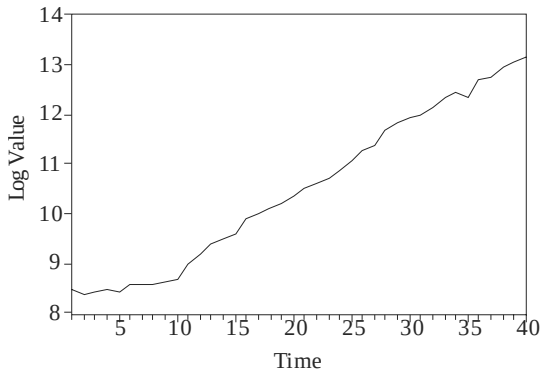
where y_t is a $(m_y \times 1)$ vector of endogenous $I(1)$ variables, x_t is a $(m_x \times 1)$ vector of exogenous variables $I(1)$, $z_t = (y', x')'$, Ψ_t is a $(d \times 1)$ vector of exogenous $I(0)$ variables excluding intercepts and time trends and t is a time trend. The symbol Δ is the difference operator and all other symbols such as a_{0y} or Π_y represent coefficients. The model assumes that there is feedback from Δy_t to Δx_t but no feedback in level, so that x_t is given as

$$\Delta x_t = a_{0x} \sum_{i=1}^{p-1} \Gamma_{ix} \Delta x_{t-i} + \zeta_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

and the disturbance ξ_t and ζ_t are distributed as an *iid*. Gaussian process with zero mean and variance Ω. Fixed public and private investment can be considered endogenous variables while the market rate of interest and change in output are exogenous. After choosing the lag length in the error correction model, Johansen trace test is used to determine the number of cointegration vector. It tests the hypothesis that number of cointegrating vector is at most equal to r.

Annual data over the span from 1964-65 to 2004-05 is used to explore the link between public and private investment. All variables, fixed public investment, fixed private investment, change in output (here output is gross domestic output), and the market rate of interest are measured as logarithms and are shown in the following four figures.[2]

Fig. 1. Private Investment



[2]Expenditures on fixed private and public investment and gross domestic product at current market prices are obtained from *50 Years of Pakistan in Statistics*, prepared by the Federal Bureau of Statistics, and *Pakistan Economic Survey* (various issues) by Finance Division. The data on national accounts has been adjusted after separation of East Pakistan by Federal Bureau of Statistics. Therefore, the study uses annual data for the period from 1964-65 to 2004-05. However, the market rate of interest (lending rate) is obtained from *International Financial Statistics (IFS)* databases developed by the International Monetary Fund (IMF).

Fig. 2. Public Investment

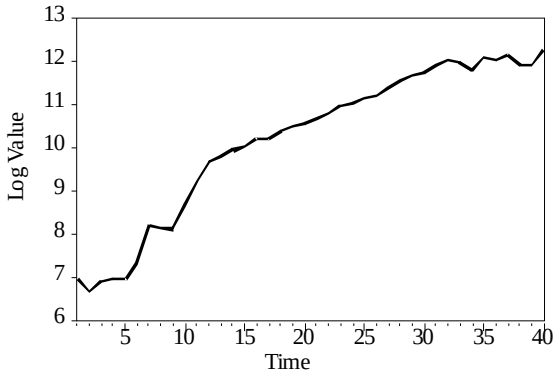


Fig. 3. The Market Rate Interest

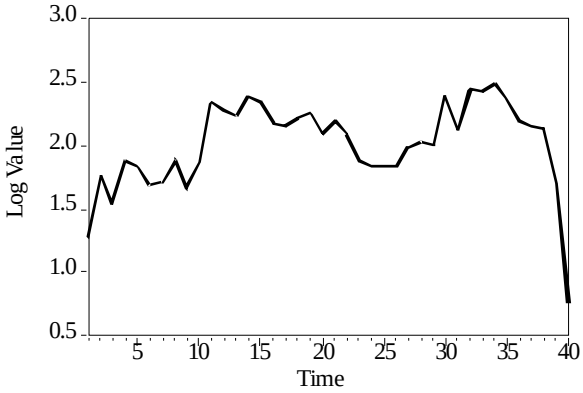
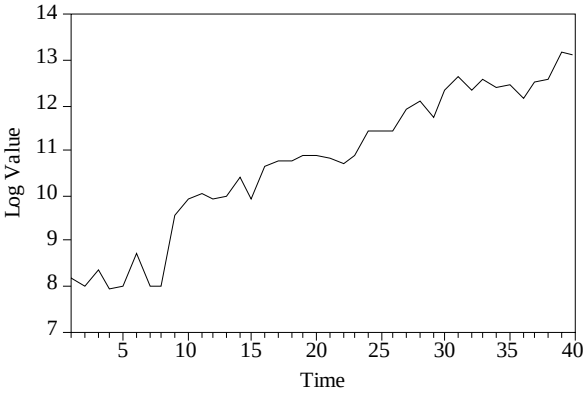


Fig. 4. Change in Output



IV. EMPIRICAL RESULTS

A number of the empirical studies in econometrics literature have reported that the classical or conventional non-stationarity test (such as DF, ADF and PP tests) is not very powerful against relevant alternatives. For instant, Delong, *et al*. (1989) found that the Dickey-Fuller tests are not able to reject a unit root null hypothesis against stable autoregressive alternatives with roots close to unity. Similarly, the study by Diebold and Rudebusch (1990) has provided empirical evidence that standard unit root tests also have low power against fractionally integrated alternatives.

To avoid this problem the present study uses the KPSS [Kwiatowski, *et al.* (1992)] methodology (the LM statistic) to test for the stationarity. Under this method, the null hypothesis is stationarity and the alternative is the presence of a unit root. This ensures that the alternative will be accepted (null rejected) only when there is strong evidence for (against) it. The KPSS test statistic is defined as follows:

$$\hat{\eta} = T^{-2}\sum \frac{S_t^2}{s^2(l)}$$

where S_t is the partial sum process of the residuals ξ_t are from a regression of the respective variable on only intercept in case of level stationary, and on intercept and trend in case of trend stationary; that is defined as $S_t = \sum_{i=1}^{t} \xi_i$ and $s^2(l) = T^{-1}\sum_{t=1}^{T} \xi_t^2 + 2T^{-1}\sum_{m=1}^{l} w(m,l) \sum_{t=m+1}^{T} \xi_t \xi_{t-m}$, $w(m,l)$ is an optional weighting function; this is, $w(m,l) = 1 - m/(1+l)$, where l is the maximum lag. The estimated tests statistics are presented in Table 1.

Table 1

KPSS Unit Root Tests

$\hat{\eta}_u$ and $\hat{\eta}_\tau$ Statistics for Level and Trend Stationarity

	Lag Truncation Parameter (l)					
	1	2	3	1	2	3
Variables	η_u : 5% Critical Value is 0.463			η_τ : 5% Critical Value is 0.146		
Private Investment	2.075	1.092	0.659	0.248	0.153	0.111
Public Investment	1.921	1.020	0.622	0.439	0.249	0.163
Change in Output	1.970	1.063	0.647	0.219	0.159	0.115
The Rate of Interest	0.319	0.217	0.164	0.187	0.134	0.105

Note: In KPSS tests, the lag orders are used to correct for error autocorrelation.

It can be seen from the table that all the said variables are non-stationary in level[3] apart from the market rate of interest that is stationary. Thus, this study considers change in output as a non-stationary exogenous and the market rate of interest as a stationary exogenous variable in Equation (1).

The next step to carry on the cointegration testing procedure is to determine the optimal lag-length and to specifying the model. To proceed with this, the Akaike Information Criterion (AIC) was calculated for lags ranging from one to four for all possible cointegration vectors form models with either restricted intercepts and no trends or unrestricted intercepts and restricted trends. The maximum absolute value of the criterion suggests a specification of model with intercept and linear trend, 2 lags and one cointegration vector. Given that it can been observed from Figure 1 to 4, the data itself exhibits strong linear trends. Therefore, these chooses seem rational and are used to explore long-run relationship between public and private investment by Johansen multivariate cointegration test.

Table 2 presents the Johansen trace test to determine the number of cointegration vectors for the specification suggested by the selection criteria. The results presented in the table strongly support the presence of one cointegration vector for public and private investment.[4]

Table 2

Johansen Cointegration Test Statistics

Hypothesis		Eigenvalue	Trace Statistics
Ho	H_A		
$r = 0$	$r > 0$	0.495	29.890
$r \geq 1$	$r > 1$	0.116	4.556

Critical values at 0.05 level are 25.32 and 12.25 for the trace test.

Normalised cointegrating coefficients with standard errors are presented in Table 3. As investment theories tell, private investment is the left-hand-side variable in Equation (1), thus the coefficient on public investment indicates a positive relationship between public and private investment. This piece of evidence is supporting the hypothesis that public investment crowds in private investment.

[3]The first differences of these variables, however, appear stationary. The estimates of KPSS tests for the first differences are not reported here to save the space, however, are available from author upon request.

[4]Trace-statistic is adjusted with degree of freedom. Thus it provides more robust results than maximum-statistic.

Table 3

Normalised Cointegrating Coefficients: 1 Cointegrating Equation

Private Investment	Public Investment	Trend	Constant
1.000	0.176	–0.116	–9.934
	(0.125)	(0.022)	

Numbers in parentheses are standard errors and Log likelihood is 66.74.

Impulse Response Function (IRF) and Variance Decompositions (VDC)

To evaluate the dynamic interactions among the variables and the relative importance of various shocks, the study uses impulse response function and variance decompositions as additional checks of the above findings. Followed by Order and Fisher (1993), Cholesk-type of contemporaneous identifying restrictions are employed to draw a meaningful interpretation. In VAR models, particularly with this type of restrictions, the results are very sensitive to ordering of the variables. The recursive structure assumes that variables appearing first contemporaneously influence the latter variables but not vice versa. It is important to list the most exogenous looking variables earlier than the most endogenous looking variables. In this study, the ordering of public investment and private investment was chosen on the basis of economic rationale to estimate the IRF and VDC. In this order, public investment has contemporaneous influence on private investment. The results of VDC are reported in Table 4.

Table 4

Variance Decomposition of Private Investment

Years Ahead	Public Investment	Private Investment
1	0.184	99.816
2	12.509	87.491
3	14.624	85.376
4	16.478	83.522

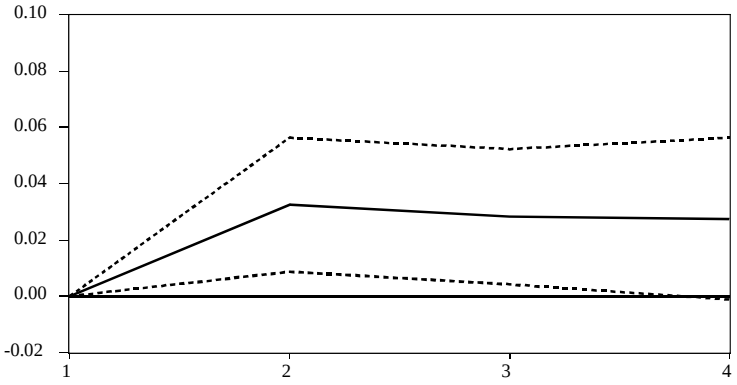
Variables are log levels as specified in the ECM, with one cointegration vector. Lag length of variables is two.

It can be observed from the tables that the VDC estimates have increasing trend, however, the size of the estimates is unexpectedly low. Accordingly, it can be concluded that public investment is relatively less important variable in explaining forecast error variance of private investment. VDC coefficients indicate the importance of variable but fail to give information about the direction of the response of variables to certain shocks. Therefore, IRF is used to examine the positive or negative response of private investment to changes in public investment.

The IRF derived from the ECM is presented in Figure 5. This function accounts the dynamic response of private investment to a one standard deviation shock of public investment. The response is considered significant if confidence

intervals do not pass through zero line. As indicated by the figure, the response of private investment to one standard deviation shock of private investment is positive and significant.

Fig. 5. Response of Private Investment[5] to One S. D Innovation ± 2 S.E.



Stationary and Heteroscedastic Cointegration

The above evidences clearly suggest one cointegration vector; however, one issue remains unsolved whether the existing cointegration is stationary or heteroscedastic. The conventional cointegration process is unable to distinguish between stationary and heteroscedastic cointegration. The fundamental difference between stationary and heteroscedastic cointegration is that the variance of a change is allowed to vary in the latter case whilst it is constant in the former. Therefore, Harris, *et al.* (2003) residual-based statistic is used to test the null of stationary cointegration against the alternative hypothesis of heteroscedastic cointegration. The test statistics is defined as follows:

$$\hat{S}_{he} = \sum_{t=1}^{T} t\xi_t^2$$

where ξ_t are OLS residuals from Equation (1). $\hat{S}_{he}$ is asymptotically normal distribution. The estimated test statistic is 3.76 which is greater than the critical values at any common level of significance. Consequently, the null hypothesis of stationary cointegration is rejected in favour of alternative hypothesis of heteroscedastic cointegration.

[5]Variables are log level and 2 lags are used to produce this result. Bands shown with dotted lines indicate one standard deviation above or below the mean. Monte Carlo simulation with hundred replications was used to compute the means of IRF and their responding standard deviations.

V. CONCLUSIONS

Multivariate cointegration approach, impulse response function and variance decompositions are used to examine the link between public and private investment for Pakistan's economy. The empirical results based on ECM show that both types of investments move together in long-run and public investment crowds in private investment.[6] The estimates of VDC provide relatively weak evidence; however, IRF indicated that the response of private investment to a shock of public investment is positive and significant. Generally, the empirical findings are providing strong support to the complementarity hypothesis.

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[6]This piece of evidence is based on aggregate data. In future, however, a study can be done to explore the relationship between public spending and private sectors' productivity using sector wise dada that may provide some useful information about the linkages between public spending and private investment.

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Comments

The paper “Public-Private Investment Linkages: A Multivariate Cointegration Analysis” deals with an important subject in the current scenario. It estimates the long-run relationship between public and private investment in Pakistan. The author must be commended for using the cointegration technique, which is a very powerful econometric instrument.

I would like to seek some clarification from the author as well as make some suggestions with reference to different sections of the paper that would help author in improving the paper.

If we look at the literature review in the paper, the author has mentioned on page 3 that the distinguishing feature of study is that it examines the link between public and private investment for a developing country rather than developed countries. It seems no such study has been conducted for developing country, as well as for Pakistan. However, to my knowledge, there are a number of such studies for developing countries. Even for Pakistan there are studies like Khan (1988), Naqivi (1993), Looney (1997) and Naqivi (2002). The respected authors should through these studies, all these are for Pakistan. Especially the Naqivi (2002) have estimated the same relationship applying same technique.

In model, on page 5 author have defined variable as public investment but does not clarify either it covers all the components of public investment, or only the components of public investment that relate to infrastructure. It is not understandable from then text.

The study mentions the use of market rate of interest, it is not clear which rate among the numerous rates available is actually used, if call money rate is used, it is not appropriate. The rate used should be theoretically appropriate.

It has been assumed that changes in output and market rate of interest are exogenous; to be sure that this assumption holds, exogeniety test should be employed. This has not been done; especially the assumption that output is exogenous is difficult to digest theoretically.

The data span is 1964-2004; no motivation for selecting the span is specified in the paper, Data of these series are available from 1959-1960. Secondly, span included encounters structural break in 1971, due to separation of East Pakistan. Econometric methodology is available to take care of such structural break in data. However, it is better to use the data from 1972 to onward to avoid structural break.

Eigen values and Trace statistics are giving conflicting conclusions about the cointegrating relationship. Maximum Eigen values test is considered powerful test of cointegration as compare to trace statistic. Eigen value test statistic reported in the

paper on page 8 shows no cointegration, where as trace test statistic indicates cointegration. It appears that author has not converted the Eigen values into maximum Eigen value. Probably the maximum Eigen values would also indicate cointegration, if the reported Eigen values are not the maximum ones.

Furthermore, the coefficient of public investment in the cointegration equation on page 8 is insignificant which shows that no relationship exists between public and private investment. This contradicts the authors, conclusions of positive relationship between the two. Secondly, author used time trend in the cointegrating equation having negative sign and highly significant, it should be explain in the text.

The author has used VDC, which is sensitive to ordering, the standard procedure is to test the results using different ordering. If the results do not change significantly with the change in ordering this implies robustness of results. The sensitivity test has not been used.

The concluding lines of the paper are that the empirical findings provide strong support to infrastructural hypothesis. It is not clear what is meant by infrastructural hypothesis. If this refers to public investment, then this assumes that all public investment is in infrastructure. This is not supported by public investment data.

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