Re-estimation of Keynesian Model by Considering Critical Events and Multiple Cointegrating Vectors

HAFSA HINA and ABDUL QAYYUM

This study employs the Mundell (1963) and Fleming (1962) traditional flow model of exchange rate to examine the long run behaviour of rupee/US $ exchange rate for Pakistan economy over the period 1982:Q1 to 2010:Q2. This study investigates the effect of output levels, interest rates and prices and different shocks on exchange rate. Hylleberg, Engle, Granger, and Yoo (HEGY) (1990) unit root test confirms the presence of non-seasonal unit root and finds no evidence of biannual and annual frequency unit root in the level of series. Johansen and Juselius (1988, 1992) likelihood ratio test indicates three long-run cointegrating vectors. Cointegrating vectors are uniquely identified by imposing structural economic restrictions on purchasing power parity (PPP), uncovered interest parity (UIP) and current account balance. Finally, the short-run dynamic error correction model is estimated on the basis of identified cointegrated vectors. The speed of adjustment coefficient indicates that 17 percent of divergence from long-run equilibrium exchange rate path is being corrected in each quarter. US war with Afghanistan has significant impact on rupee in short run because of high inflows of US aid to Pakistan after 9/11. Finally, the parsimonious short run dynamic error correction model is able to beat the naïve random walk model at out of sample forecasting horizons.

JEL Classification: F31, F37, F47

Keywords: Exchange Rate Determination, Keynesian Model, Cointegration, Out of Sample Forecasting, Random Walk Model

1. INTRODUCTION

Stability of exchange rate is crucial for economic development. It provides the macroeconomic links among the countries via goods and asserts market [Moosa and Bhatti (2009)]. In literature different approaches have been developed to analyse the behaviour of exchange rate. Among them, purchasing power parity (PPP) is the earliest approach for exchange rate determination, introduced by Swedish economist Gustav Cassel in 1920s. Empirical evidence of PPP theory has been rather mixed, In case of Pakistan, for example, Chisti and Hasan (1993) do not support PPP model to explain the exchange rate variations. Bhatti and Moosa (1994) argued that the failure of PPP under flexible exchange rate is due to the negligence of expectations in exchange rate determination. Bhatti (1997) investigated and proved the ex-ante version of PPP, in
which exchange rate is explained not only by current relative prices but also by the expected real exchange rate. Moreover, Bhatti (1996), Qayyum, et al. (2004) and Khan and Qayyum’s (2008) results do support the validity of relative form of PPP in Pakistan.

PPP theory is based on the concept of good arbitrage and ignores the importance of capital movements in exchange rate determination. To fill this gap Keynesian approach of exchange rate determination is initiated by introducing the capital flows into current account balance of payment approach [Mundell (1962) and Fleming (1962)]. The empirical validity of this structural model is tested by Bhatti (2001) for determining Pakistani rupee exchange rates against six industrial countries’ currencies. He suggested that nominal exchange rate of Pakistan is determined by relative price level, relative income level and interest rates differentials. The relative version of exchange rate model assumes symmetry in the coefficients of domestic and foreign coefficients. However, no former information is available to assume this symmetry. Moreover, relative version of exchange rate models is unable to find the multiple cointegrating vectors. Multiple cointegrating vectors contain valuable information and should be carefully interpreted [Dibooglu and Enders (1995)]. In international literature a lots of studies are available that established and uniquely identified the multiple cointegrating vectors [see for example, Juselius (1995); Dibooglu and Enders (1995); Helg and Serati (1996); Diamondis, et al. (1998); Cushman (2007); Tweneboah (2009) among others]. This study, therefore, considers the non-relative version of Keynesian exchange rate and test the symmetry among the domestic and foreign price level, output level and the interest rate. Keynesian model also incorporates the uncovered interest parity (UIP) and purchasing power parity (PPP) conditions. The identification of these parity conditions are also the aim of this paper.

One of the objectives of structural exchange rate models, like Keynesian flow model, is to explain the exchange rate variations and provide better forecast. In this regard, literature on exchange rate forecasting is divided into two categories. One which emphasises the importance of economic theory for exchange rate prediction and recommends a theory based on plausible channel to stabilise it [see, Khalid (2007); Abbas, et al. (2011)]. Similarly, Cushman (2007) empirically tested the out of sample forecasting performance of dynamic portfolio balance model of exchange rate with benchmark random walk by adopting Mark (1995) technique. On the basis of Root Mean Square Error (RMSE) and Diebold-Mariano (DM) test he suggested that structural model outperforms the random walk models at longer horizons. Likewise, MacDonald (1997), Hwang (2001), Korap (2008), and Anaraki (2007) have used multilateral cointegration technique and presented the superiority of fundamental models over random walk models. Cheung, et al. (2002) documented that the better performance of structural models are credited to the dynamic error correction model with stochastically varying coefficients and recursively updating the long run cointegrating vectors. On the other hand the promoters of random walk model argued that exchange rate is a random walk phenomenon. It efficiently analyses the exchange rate fluctuations and provides better future forecast such as Rashid (2006) and Malik (2011). According to these studies there is no need to worry about the macroeconomic variables of exchange rate determination. Meese and Rogoff (1983) and Najand and Bond (2000) suggested that the poor performance of structural models is characterised by unstable parameters. The stability of parameters is usually disturbed by the existence of outlier in the series. Therefore, it is
necessary to control the outliers in order to get better forecast [Balke and Famby (1994) and Dijk, et al. (1999)]. Therefore, to judge the out of sample forecasting performance of the dynamic error correction model of Keynesian model as compared to naïve random walk model is the other objective of this paper.

Brief overview of exchange rate systems confirms that currencies under flexible exchange rate system generally tend to depreciate more than currencies having fixed exchange rate system due to the occurrence of critical events [Ltaifa, et al. (2009)]. Pakistan had adopted a flexible exchange rate system since 2000 and its currency is freely floating against US dollar. Therefore, any shocks in US economy directly hit the Pakistan rupee. After 2001, nominal exchange rate of Pakistan is highly volatile, though, the other economic fundamentals remain the same. Its instability is attached to the happening of critical events during this era. 9/11 event and US war against terror in Afghanistan had appreciated the rupee against US dollar. This appreciation was driven by high inflows of remittances and foreign capital inflows into Pakistan. The trend of the appreciation of rupee was reversed into depreciation when Global Financial Crisis (GFC) occurred in 2007. In the period of GFC the foreign exchange reserves declined from $14.2 billion in 2007 to $3.4 billion in 2008. Pakistan rupee against US dollar lost its value by 21 percent during 2008. So far no study is available to test the significance of these critical events on the exchange rate in the framework of Keynesian model. This paper fills this gap by examining the effect of critical events on the exchange rate of Pakistan in terms of intervention dummies.

The rest of the study is organised as follows: Section 2 presents the theoretical framework of Keynesian model. Section 3 deals with the econometric methodology. Data and construction of variables is subject of Section 4. Section 5 describes the empirical results and Section 6 reports the out of sample forecasts. Section 7 concludes the study and identifies some policy implications.

2. THEORETICAL FRAMEWORK

The traditional Keynesian approach is developed by Mundell (1962) and Fleming (1962). They extended the Keynesian IS–LM framework to an open economy by incorporating the capital flows via balance of payments.

The objective of this section is to derive the reduced form equation of the equilibrium exchange rate under the Keynesian approach. In the literature a number of studies, for example Gylfason and Helliwell (1983), Pearce (1983), Bhatti (2001) and Moosa and Bhatti (2009), have derived the Keynesian equilibrium exchange rate model by utilising BOP Equation (1)

$$\Delta f = CA \left( \frac{SP^f}{P}, Y^f, Y \right) + K \left( i, i^f \right) \ldots \ldots \ldots \ldots (1)$$

Equation (1) defines the balance of payments. $\Delta f$ denotes the change in foreign reserves which equals zero under the flexible exchange rates. Current account ($CA$) is positively related to real exchange rate ($\frac{SP^f}{P}$), where $S$ denotes nominal exchange rate measured by domestic currency per unit of foreign currency, $P$ represents domestic prices and $P^f$ the
foreign price level. An increase in foreign output ($Y^f$) and depreciation of domestic currency has favourable effect on the balance of trade (BOT) by enhancing the demand for domestic exports. However, it deteriorates due to an increase in domestic output level ($Y$).

The traditional flow model also assumes that foreign and domestic assets are imperfect substitutes, which implies that interest rate differentials may cause finite capital flows into or out of a country. Thus, the net capital inflow ($K$) is a positive function of domestic interest rate ($i$) and negative function of foreign interest rate ($i^f$).

To derive the fundamental equation of exchange rate, the BOP, Equation (1) can be written as:

\[
BOP = a\left(\frac{SP^f}{P}\right) + b^f Y^f - b Y + c^f i^f
\]

All variables of Equation (2) except interest rate are in logarithm form and denoted by small letters. For simplicity a restriction $\beta^f = \beta$ and $\alpha^f = \alpha$ is imposed. The equilibrium exchange rate is determined when BOP is in equilibrium i.e. the net of current and capital account is zero and solving for nominal exchange rate $'s'$, we have

\[
s = (p - p^f) + \frac{b}{a}(y - y^f) - \frac{c}{a}(i - i^f)
\]

which explains that the equilibrium exchange rate is positively related to relative prices and relative incomes, but inversely related to relative interest rates. In general form, the above Equation (3) is written as:

\[
s = f(p, p^f, y, y^f, i, i^f)
\]

MacDonald (1995) defined the theory of long-run exchange rate modeling by relating the concepts of uncovered interest rate parity, absolute and efficient markets PPP to a standard balance of payments equilibrium condition. In order to link the absolute PPP with the current account balance he asserted that under a long-run net capital flows were zero when savings were at their desired level. This specification reduces the BOP account to current account balances. Thus we can write the Equation (3) as:

\[
s = (p - p^f) + \frac{b}{a}(y - y^f)
\]

The current account balance approaches to PPP only when the difference between domestic and foreign income level i.e. ($y - y^f$) tends to be zero. This would be possible if the price elasticity of domestic exports is infinitely large ($a \rightarrow \infty$) [MacDonald (1995) and Moosa and Bhatti (2009)], in this case the exchange rate is exclusively determined by the PPP that is:

\[
s = (p - p^f)
\]

On the other hand, the non-zero value of ($y - y^f$) is likely to be most important when comparing countries at different stages of development, but less important for countries at a similar level of development. Allowing a constant in Equation (6) would represent a permanent deviation from absolute PPP due to productivity differentials and other factors [MacDonald (1995) and Taylor and Taylor (2004)].
The efficient market view of PPP suggests that in a world of high or perfect capital mobility it is not goods arbitrage that matters for the relationship between an exchange rate and relative prices, but interest rate arbitrage. Hence, a slow speed goods market arbitrage causes a temporary deviation of the exchange rate from PPP. This requires that the exchange rate drifts in such a manner as to restore the relative PPP. Algebraically these deviations can be expressed as:

$$\Delta s = p - p^f - s \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$ (7)

A perfectly mobile capital immediately diverts the attention to focus on the capital account of the balance of payments. The assumption of perfect capital mobility may be represented as:

$$\Delta s^e = i - i^f \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$ (8)

Equation (8) represents the uncovered interest parity condition. This condition defines that the difference between the domestic interest rate ($i$) and foreign interest rate ($i^f$) produces an expected depreciation of the exchange rate. Frenkel (1978) and Juselius (1995) among others, argued that the fluctuations in exchange rate are attributed by both goods and assets market development. Therefore, PPP and UIP conditions may not be independent of each other in the long run. This allows us to substitute Equation (8) into Equation (7) to combine PPP with UIP and model the nominal exchange rate as:

$$s = p - p^f - i + i^f \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$ (9)

The above discussion makes it clear that it is not worthwhile to empirically analyze the short run relationship between exchange rate, domestic and foreign price level, interest rate and output and ignore their long run associations (defined in Equations (5) to (9)). Hence, long run relationship(s) would be combined with the short run dynamics of exchange rate by employing the vector error correction mechanism.

### 3. EMPIRICAL METHODOLOGY

#### 3.1. Unit Root Test

Cointegration analysis is based on the assumption that variables are integrated of same order. Pre-testing for unit root is necessary to avoid the problem of spurious regression. Hylleberg, Engle, Granger, and Yoo (HEGY) (1990) is used to test for non-seasonal zero frequency unit root and biannual and annual frequency seasonal unit roots in quarterly data. HEGY provide following auxiliary regression equation:

$$\Delta_4 y_t = \mu + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{4,t-2} + \sum_{i=1}^3 \gamma_i \Delta_4 y_{t-i} + \epsilon_t \quad \ldots$$ (10)

Where $\mu_t$ is a deterministic term which can include any combination of a drift term, trend term and a set of seasonal dummies, $y_{1,t}, y_{2,t}, y_{3,t},$ and $y_{4,t}$ are linearly transformed series as proposed by HEGY i.e., $y_{1,t} = (1 + B)(1 + B^2) y_t, y_{2,t} = -(1 - B)(1 + B^2) y_t, y_{3,t} = -(1 - B) (1 + B) y_t$ and $y_{4,t} = (1 - B^4) y_t$, where $B$ is a lag operator such that $B^k y_t = y_{t+k}, \epsilon_t \sim (0, \sigma^2_\epsilon)$ is...
Gaussian error term and white noise $Cov(e_t, e_{t-1}) = 0$. The auxiliary regression (10), comes from the fact that $\Delta_t = (1 - B^4)$ can be decomposed as $(1 - B)(1 + B)(1 \pm iB)(1 \mp iB)$ where each term in bracket corresponds to non-seasonal zero frequency unit root, biannual frequency unit root $1$ and annual frequency unit root $i$.

HEGY method tests the significance of $\pi_j$ (j=1,2,3,4) parameters. If $\pi_1 = 0$ is statistically significant then series contain non-seasonal zero frequency unit root. If $\pi_2 = 0$ is accepted this implies the presence of biannual frequency seasonal unit root. If $\pi_3 = \pi_4 = 0$, then series has seasonal unit root at annual frequency. The appropriate filter corresponding to the acceptance of each null hypothesis are $(1-\Delta)$, $(1+B)$ and $(1+B^2)$ required to make the series stationary. Critical values for one sided t-test for $\pi_1 (t_{s1})$, $\pi_2 (t_{s2})$ and for the joint F-test for $\pi_3$ and $\pi_4 (F_{3,4})$ are provided by HEGY.

3.2. Johansen and Juselius Cointegration Methodology

Johansen and Juselius (1990) cointegration technique is useful to construct a multiple long-run equilibrium relationships over multivariate system. Generally, this technique is applied to $I(1)$ variables. Johansen’s method in $k$ dimensional error correction (EC) form is presented as follows:

$$\Delta z_t = \sum_{i=1}^{l-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-i} + \phi D_t + \mu + \epsilon_t \ldots \ldots \ldots \ldots (11)$$

Where $z_t$ is $(k \times 1)$ dimensional vector of $I(1)$ variables, $D_t$ consists of centered seasonal dummies, intervention and policy dummies such that all are $I(0)$, $\mu$ is deterministic trend component, which consist of different combinations of constant and trend terms in the long-run cointegrating equation and short-run vector auto regressive (VAR) model, $\epsilon_t \sim N(O, \Sigma)$ is $(k \times 1)$ vector of Gaussian random error terms and $\Sigma$ is $(k \times k)$ variance covariance matrix of error terms. $(i = 1, 2, \ldots, l - 1)$ is the lag length. $\Gamma_i = -(I - A_1 - \ldots \ldots - A_l)$ is short-run dynamic coefficients. $\Pi = -(I - A_1 - \ldots \ldots - A_k)$ is $(k \times k)$ matrix containing long-run information regarding equilibrium cointegration vectors.

The number of cointegrating vectors $(r)$ are determined by rank of $\Pi$ matrix. If $0 < rank(\Pi) < k - 1$ then it is further decomposed into two matrices i.e. $\Pi = \alpha \beta^\prime$: $\alpha$ is a $(k \times r)$ matrix containing error correction coefficients, which measure the speed of adjustment to disequilibrium. $\beta^\prime$ is $(r \times k)$ matrix of $r(\Pi)$ cointegrating vectors. The rank of $\Pi$ matrix is measured by likelihood ratio trace and maximum eigenvalue statistics. In case of multiple cointegrating vectors Johansen and Juselius (1990) allow the imposition of linear economic restrictions on $\beta$ matrix to obtain long-run structural relationships.

3.3. Short-Run Dynamic Error Correction Model

According to Granger (1983) Representation Theorem, if there is long-run stable relationship among the variables then there will be a short-run error correction relationship related with it. Short-run vector error correction representation is as follows:
\[ \Delta z_t = \sum_{i=1}^{l-1} \Gamma_i \Delta z_{t-i} + \alpha (\beta' z_{t-1}) + \phi D_t + \mu + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad (12) \]

\( \beta' z_{t-1} \) is the error correction term. The traditional methodology uses the residuals from the identified cointegrating vector(s) to form \( \beta' z_{t-1} \). \( \alpha \) in dynamic error correction model measures the speed of adjustment toward equilibrium state. Theoretically speed of adjustment coefficient must be negative and significant to confirm that long-run relationship can be attained.

4. DATA AND CONSTRUCTION OF VARIABLES

This study considers quarterly data from 1982:Q1 to 2010:Q2. A start from 1982 is on account of implementation of flexible exchange rate policy in Pakistan. All variables are measured in the currency units of each country. The data are obtained from International Financial Statistics (IFS) and State Bank of Pakistan (SBP) Monthly Statistical Bulletin (Various Issues).

The nominal exchange rate is measured in terms of Pakistan rupee (PKR) per unit of US dollar (US $). Real Gross domestic product (GDP) is commonly used as a measure of real output level. Quarter wise real GDP of US is accessible from IFS. In case of Pakistan only annual real GDP is available. Quarterisation of annual real GDP is done by using the methodology of Kemal and Arby (2004). Consumer price index (2000=100) is used as a proxy of domestic and foreign price level. Call money rate for Pakistan and federal fund rate for US are used as a measure of interest rates. During the analysis period exchange rate of Pakistan is also influenced by the critical events such as 1998 Pakistan’s nuclear test, 9/11 event, US war against terror in Afghanistan after 9/11, 2005 stock market crash and recent global financial crisis (2007). Dummy variables \( D_{98} \) (0 for \( t < 1998: Q2 \) and 1 for \( t 1998: Q2 \)), \( D_{911} \) (1 for \( t = 2001: Q3 \) and 0 otherwise), \( D_{afgwar} \) (0 for \( t < 2001: Q4 \) and 1 otherwise), \( D_{SMC} \) (1 for \( t = 2005: Q1 \) and 0 otherwise) and \( D_{fc} \) (0 for \( t < 2007: Q1 \) and 0 otherwise) are used to capture the influence of these events on the exchange rate.

5. RESULTS AND DISCUSSION

This section implements the Johansen and Juselius (1988, 1992) multivariate cointegration methodology to detect the stable long run relationships between the exchange rate and fundamental variables. The preliminary time series properties for cointegration analysis are as follows:

5.1. Order of Integration (Unit Root Test)

The presence of seasonal and non-seasonal unit roots for each quarterly series is determined via HEGY (1990) test. All variables are transformed in logarithmic form except the interest rate. The results of the HEGY test are presented in Table 1. It can be observed that the null hypothesis of a non-seasonal unit root cannot be rejected whereas the null hypothesis of seasonal unit root at both biannual and annual frequency are rejected at 5 percent critical values for all of the variables. (1-B) is an appropriate filter to make the series stationary. The results of HEGY test after applying required filter are
presented in Table 2 and we found no evidence of seasonal and non-seasonal unit roots at 5 percent level of significance. Therefore, all variables in our cointegration analysis are integrated of order one and we may suspect multiple long run cointegrating vectors.

Table 1

HEGY Regression Model

\[ \Delta y_t = \alpha + \beta t + \sum_{i=1}^{4} \delta_i y_{t-i} + \pi_1 y_{t-1} + \pi_2 y_{t-2} + \pi_3 y_{t-3} + \pi_4 y_{t-4} + \sum_i \Delta y_{t-i} + \epsilon_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>Drift</th>
<th>Trend</th>
<th>Seasonal Dummies</th>
<th>Test Statistic</th>
<th>Null &amp; Alternative Hypothesis</th>
<th>Roots (Filter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-0.81</td>
<td>( \pi_1 = 0 ) ( \pi_2 = 0 ) ( \pi_3 = \pi_4 = 0 )</td>
<td>1 (1-B)</td>
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<tr>
<td>( Y )</td>
<td>3</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-1.10</td>
<td>( \pi_i &lt; 0 ) ( \pi_i &lt; 0 ) ( \pi_i \neq \pi_4 \neq 0 )</td>
<td>1 (1-B)</td>
</tr>
<tr>
<td>( y' )</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-3.06</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
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</tr>
<tr>
<td>( P )</td>
<td>0</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>-1.69</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>1 (1-B)</td>
</tr>
<tr>
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<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-2.46</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
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<td>( i )</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>-0.23</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
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<tr>
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<td>Yes</td>
<td>Yes</td>
<td>-3.14</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>1 (1-B)</td>
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Table 2

HEGY Test on Filtered Series

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>Drift</th>
<th>Trend</th>
<th>Seasonal Dummies</th>
<th>Test Statistic</th>
<th>Null &amp; Alternative Hypothesis</th>
<th>Roots</th>
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<tr>
<td>( (1-B) s )</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-4.86</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>-</td>
</tr>
<tr>
<td>( (1-B) y )</td>
<td>2</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-2.96</td>
<td>( \pi_i &lt; 0 ) ( \pi_i &lt; 0 )</td>
<td>1</td>
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<tr>
<td>( (1-B) y' )</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-3.69</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>-</td>
</tr>
<tr>
<td>( (1-B) P )</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-3.07</td>
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<tr>
<td>( (1-B) p' )</td>
<td>0</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>-4.34</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>-</td>
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<td>( (1-B) i )</td>
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<td>No</td>
<td>No</td>
<td>No</td>
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<td>( \pi_i = 0 ) ( \pi_i = 0 )</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>-4.94</td>
<td>( \pi_i = 0 ) ( \pi_i = 0 )</td>
<td>-</td>
</tr>
</tbody>
</table>

5.2. Unrestricted VAR Model Specification

The next step after implementing the unit root test is to decide the optimal lag length of the multivariate system of equations, which ensures that residuals of VAR model are white noise. We have used Johansen (1995) multivariate LM test and 3 quarters have been selected as appropriate lag structure of the model. Three central seasonal dummies and four intervention dummies \( D_{99}, D_{901}, D_{adgwar}, D_{1c} \) are also included. The residual of the VAR(3) passed the diagnostic test of no serial correlation \( (\chi^2_{(49)}=52.31) \) with four lags), no heteroscedasticity \( (\chi^2_{(1372)}=1355.36) \) at 5 percent level of significance, but fail to pass the null hypothesis of normally distributed error terms under Jarque-Bera (JB) test \( (\chi^2_{(14)}=73.24) \). However, lack of normality does not affect the

5.3. Multivariate Cointegration Analysis

After selecting the lag length of VAR model, another fundamental issue is the suitable treatment of deterministic components such as drift and trend terms in the cointegrating and the VAR part of the VECM. Most of the series in our analysis exhibit a linear trend in the level of the series. Therefore, we introduce intercept term unrestrictedly both in long run (cointegrating part) and short run (VAR) model while performing cointegration analysis [Johansen (1995); Harris, et al. (2003) and Qayyum (2005)]. Table 3 presents the trace and maximum eigenvalue statistic after adjusting by factor (T-kl)/T to correct the small sample bias.

Table 3

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Chi-Square</th>
<th>0.05 Critical Value</th>
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<tbody>
<tr>
<td>Trace Statistic</td>
<td></td>
<td></td>
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<tr>
<td>r = 0</td>
<td>r &gt; 0</td>
<td>155.05*a</td>
<td>125.62</td>
</tr>
<tr>
<td>r ≤ 1</td>
<td>r &gt; 1</td>
<td>104.24*a</td>
<td>95.75</td>
</tr>
<tr>
<td>r ≤ 2</td>
<td>r &gt; 2</td>
<td>71.43*a</td>
<td>69.82</td>
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<tr>
<td>r ≤ 3</td>
<td>r &gt; 3</td>
<td>40.78</td>
<td>47.86</td>
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<td>r ≤ 4</td>
<td>r &gt; 4</td>
<td>20.94</td>
<td>29.80</td>
</tr>
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<td>r ≤ 5</td>
<td>r &gt; 5</td>
<td>5.77</td>
<td>15.49</td>
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<td>r ≤ 6</td>
<td>r &gt; 6</td>
<td>0.29</td>
<td>3.84</td>
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<td>Maximum Eigenvalue Statistic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r = 0</td>
<td>r = 1</td>
<td>50.81a</td>
<td>46.23</td>
</tr>
<tr>
<td>r = 1</td>
<td>r = 2</td>
<td>32.81</td>
<td>40.08</td>
</tr>
<tr>
<td>r = 2</td>
<td>r = 3</td>
<td>30.65</td>
<td>33.88</td>
</tr>
<tr>
<td>r = 3</td>
<td>r = 4</td>
<td>19.85</td>
<td>27.58</td>
</tr>
<tr>
<td>r = 4</td>
<td>r = 5</td>
<td>15.16</td>
<td>21.13</td>
</tr>
<tr>
<td>r = 5</td>
<td>r = 6</td>
<td>5.49</td>
<td>14.26</td>
</tr>
<tr>
<td>r = 6</td>
<td>r = 7</td>
<td>0.29</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Note: ‘a’ indicates the rejection of null hypothesis at the 5 percent level.

The trace test shows that the null hypothesis of no cointegration (r=0), one cointegration (r ≤ 1) and two cointegrating vectors (r ≤ 2) can be rejected, but fails to reject the null of three cointegrating vectors at 5 percent level of significance. Therefore, variables of Keynesian exchange rate model are found to be cointegrated with three cointegrating vectors. Whereas, the maximum eigenvalue statistic with the null hypothesis r=1 is rejected, but the null hypothesis of r=2 is not rejected and refers to one long run relationship among the variables.¹ This contradiction among the tests for cointegrating vector is common. We continue our analysis on the basis of trace test, as it

¹Before adjusting trace test reports four while maximum eigenvalue test indicates three cointegrating equations at 5 percent level of significance (results are not presented here).
is a more powerful test as compared to maximum eigenvalue statistics in case of not normally distributed error terms [Cheung and Lai (1993); Hubrich, et al. (2001)]. Kasa (1992) and Serletis and King (1997) also preferred trace statistics as it considers all k-r (k is no. of variables in the system and r is the cointegrating vectors) values of smallest eigenvalues.

The first three cointegrating vectors with the maximum eigenvalue have been normalised on log of nominal exchange rate to determine the sign and magnitude of the long-run elasticities in Keynesian exchange rate model Equation (4). The results of normalised vectors are presented in Equation (13);

\[ s_1 = 270.69 + 1.06 p_t + 20.77 p_t^f + 4.73 y_t - 26.89 y_t^f - 0.40 i_t + 0.55 i_t^f \]
\[ s_2 = -279.51 + 1.01 p_t - 23.67 p_t^f - 3.84 y_t + 27.39 y_t^f + 0.53 i_t - 0.73 i_t^f \]
\[ s_3 = -97.67 + 1.16 p_t - 1.32 p_t^f - 9.62 y_t + 14.68 y_t^f - 0.06 i_t - 0.04 i_t^f \] … (13)

Result shows that the sign of all variables except the foreign price level are consistent with Keynesian theory in first cointegrating vector. In second cointegrating vector the signs of domestic and foreign price level, while, in the third vector the signs of domestic price level, foreign price level and domestic interest rate support the theory. The contradiction of results among the vectors arises due to arbitrary normalisation. It restricts to draw a meaningful conclusion.

As described earlier that multiple cointegrating vectors contain valuable information and must be identified properly and carefully interpreted. To obtain this information we start by imposing proportionality and symmetry restrictions on all vectors in proceeding section.

5.4. Proportionality and Symmetry Restrictions

Before the identification of cointegrating vectors, we proceed to test the proportionality and symmetry restrictions of prices, interest rates and output through likelihood ratio test on all cointegrating vectors. The acceptance of these restrictions provides the validity of strict form PPP and UIP. The likelihood ratio (LR) test statistics along with their probability values for the proportionality and symmetry restrictions are reported in Table 4.

First part of Table 4, reports the results of symmetry restrictions on prices, output and interest rates on all three cointegrating vectors in order to find whether they enter in the equilibrium relation or not. The symmetry restriction implies that prices, output and interest rates influence the exchange rate regardless of where they originate. According to LR test statistics, symmetry restrictions hold for prices and output. Under H3, we found no evidence of interest rate symmetry. The joint symmetry restrictions implied by H4 through H7 are mostly rejected at 95 percent level of significance.

Further, the proportionality restriction (H8) holds for prices but not for output and interest rate in all three cointegrating vectors. Symmetry and proportionality of prices is opposite to the finding of Khan and Qayyum (2008). The basic reason for this contradiction is the absence of other fundamental variables such as output levels and interest rate in their analysis. In our analysis we can predict the long run strong form PPP in the presence of other fundamental variables.
Table 4

Restricted Cointegrating Vectors

\[ s_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p_{t-1} + \alpha_3 y_t + \alpha_4 y_{t-1} + \alpha_5 d_t + \alpha_6 d_{t-1} + u_t \]

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Restrictions</th>
<th>( \chi^2 ) (df)</th>
<th>P- Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Symmetry Restrictions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Symmetry</td>
<td>( H_1: \alpha_1 = -\alpha_2 )</td>
<td>9.33(3)*</td>
<td>0.03</td>
</tr>
<tr>
<td>Output Symmetry</td>
<td>( H_2: \alpha_3 = -\alpha_4 )</td>
<td>7.13(3)**</td>
<td>0.08</td>
</tr>
<tr>
<td>Interest Rate Symmetry</td>
<td>( H_3: \alpha_5 = -\alpha_6 )</td>
<td>16.41(3)</td>
<td>0.00</td>
</tr>
<tr>
<td>Price and Output Symmetry</td>
<td>( H_4: H_1 \cap H_2 )</td>
<td>15.73(6)*</td>
<td>0.02</td>
</tr>
<tr>
<td>Price and Interest Rate Symmetry</td>
<td>( H_5: H_1 \cap H_3 )</td>
<td>23.24(6)</td>
<td>0.00</td>
</tr>
<tr>
<td>Output and Interest Rate Symmetry</td>
<td>( H_6: H_2 \cap H_3 )</td>
<td>23.00(6)</td>
<td>0.00</td>
</tr>
<tr>
<td>Joint Symmetry of Prices, Interest Rate and Output</td>
<td>( H_7: H_1 \cap H_2 \cap H_3 )</td>
<td>25.92(9)</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Proportionality Restrictions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_8: \alpha_1 = \alpha_2 = 1 )</td>
<td>14.80(6)*</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>( H_9: \alpha_3 = \alpha_4 = 1 )</td>
<td>32.85(6)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>( H_{10}: \alpha_5 = \alpha_6 = 1 )</td>
<td>32.85(6)</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: *, **, and *** indicate the significance at 1 percent, 5 percent and 10 percent.

5.5. Identification of Cointegrating Vectors

In Table 5, we proceed by imposing the theoretical restrictions on PPP, UIP and their combinations. First part of Table 5 reports individual parity conditions. Under \( H_{11} \), strict version of PPP is tested in all cointegrating vectors. The LR test statistics for this hypothesis yields to accept the strong form of PPP with other fundamental variables at 10 percent level of significance. Similarly strong PPP form with unrestricted output coefficients (\( H_{24} \)) and with unrestricted interest rate coefficients (\( H_{22} \)) are also accepted at 5 percent level of significance.

\( H_{12} \) analysed the strict form of PPP in the first cointegrating vector. This was done by executing unity restriction on exchange rate and prices and zero restriction on output and exchange rates coefficients in the first cointegrating vector. This hypothesis is rejected by LR test. This result suggests that strong form of PPP does not hold on its own. Weak form of PPP is investigated under \( H_{13} \) and \( H_{14} \), both of these hypothesis are rejected by LR test.

The rejection of both strict and weak forms of PPP on its own (in the absence of other fundamental variables) is consistent with Khan and Qayyum (2008), Helg and Serati (1996), Dibooglu and Enders (1995) and Macdonald (1993). Last two authors argued that this is due to the different ways of finding national indices, which result into the non proportionality of price adjustments. According to Helg and Serati (1996), standard PPP does not hold on its own during the period of flexible exchange rate. Khan and Qayyum (2008) argue that rejection of strong form of PPP is due to the significance of transportation and transaction cost.
<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Restricted CI vectors ( s \quad p \quad p' \quad y \quad y' \quad i \quad i' )</th>
<th>( \chi^2 ) (df)</th>
<th>P- Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individual Parity Conditions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| PPP in all Three Vectors (Strict PPP with other Fundamental Variables) | \( H_{11} \): \( 1 \quad -1 \quad 1 \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \quad * \ quadrant of the table. 

*Note: * In column three represents no restriction.  
\( ^{a} \), \( ^{aa} \), and \( ^{aaa} \) in column four indicate the significance at 1 percent, 5 percent and 10 percent.
After investigating the different versions of PPP restrictions, we now analyse the UIP condition. First we examine whether strong form of UIP restriction enters in all three cointegrating vectors or not, by formulating \( H_{15} \). This hypothesis is strongly rejected by LR test. However, under \( H_{16} \), we set out that UIP relationship is stationary by itself by imposing unity restriction on interest rate coefficients and zero restriction on prices and output coefficients in first cointegration vector. The LR test result supports that one of the cointegrating vectors contains a stationary relationship between the interest rate variables. This result is consistent with Johanson and Juselius (1992).

Further, the weak form of UIP is tested in all cointegrating vectors by \( H_{17} \) and in first cointegrating vectors through \( H_{18} \) with zero restriction on the coefficient of prices and output. \( H_{17} \) is rejected by the LR test, whereas, the later hypothesis is not rejected by LR test. From the results of various forms of UIP conditions, we can conclude that UIP holds without the fundamental variables in one cointegrating vector only.

Following this, we combined PPP and UIP restrictions by \( H_{19} \) through \( H_{23} \). On the basis of LR statistic the strong form of PPP along with strong form of UIP (\( H_{19} \)), weak form of PPP with strong form of UIP (\( H_{21} \)) and strong form of PPP with weak UIP (\( H_{22} \)) enter in the cointegrating vector.

Finally the joint hypothesis of PPP, UIP and output symmetry in one cointegrating vector is not rejected under \( H_{27} \).

The general hypothesis tested through \( H_1 \) to \( H_{27} \), are informative to formulate unique vectors in the multiple cointegration space. These results suggest that strong form of PPP with output relationship (\( H_{23} \)) is considerable in one vector while the weak form of UIP relationship (\( H_{18} \)) is in the second vector and the strict form of PPP and unrestricted interest rate is in the third vector. All cointegrating vectors are normalised on nominal exchange rate. Thus, it would seem plausible to specify the long run cointegrating vector \( \beta' \) matrix as:

\[
\beta' = \begin{bmatrix}
1 & -1 & 1 & * & * & 0 & 0 \\
1 & 0 & 0 & 0 & * & * \\
1 & -1 & 1 & 0 & 0 & * & *
\end{bmatrix}
\]

The LR test statistics for these restrictions are \( \chi^2_{df=6} = 11.88 \), which do not reject this hypothesis. The results of long-run cointegrating vectors are presented as:

\[
s_t = p_t - p_t^* + 6.37y_t - 8.66y_t^* + 55.06 \quad \ldots \quad \ldots \quad \ldots (14)
\]

\[
s_t = 0.65i_t - 0.76i_t^* + 2.06 \quad \ldots \quad \ldots \quad \ldots \quad \ldots (15)
\]

\[
s_t = p_t - p_t^* + 0.19i_t - 0.24i_t^* + 3.47 \quad \ldots \quad \ldots \quad \ldots (16)
\]

The results of restricted vectors suggest that exchange rate is determined by both current account balance and net capital inflows. The estimated signs of all variables except the domestic and foreign interest rates are consistent with Keynesian theory. On the basis of cointegrating vectors following results can be made:
Strong form of PPP does not hold on its own but holds with other fundamental variables. This result supports the arguments by Helg and Sarati (1996) and Khan and Qayyum (2008) i.e., the rejection of strong form of PPP on its own is due to the significance of transportation and transaction cost. However, increase in domestic (foreign) price level will lead to depreciation (appreciation) of the domestic currency.

Positive (negative) coefficient of domestic (foreign) output reveals that increase in domestic (foreign) output level results in depreciation (appreciation) of domestic currency via higher demand of imported (exported) commodities. Hence, stronger economic growth of Pakistan tends to cause depreciation in the exchange rate. This is because the growth is led by higher consumer spending, this will cause a rise in imports which could lower the exchange rate.

Positive impact of domestic interest rate on exchange rate suggests that increase in domestic interest rate leads to depreciation of the domestic currency against US dollar. Whereas, increase in foreign interest rate results in the appreciation of domestic currency. The estimated coefficients of both interest rates are not according to the theory, the opposite signs of interest rates were also observed in Bhatti (2001).

5.6. The Short-Run Function for Keynesian Exchange Rate: Dynamic Error Correction Model

This section presents the short-run dynamic error correction model (ECM) of the Keynesian exchange rate model. The residuals of the long run cointegration functions (from Equations 14 to 16) are used as an important determinant of ECM. These residuals are also known as disequilibrium estimates or error correction terms. They measure the divergence from long run equilibrium in period $t-1$ and provide the speed of adjustment information toward equilibrium.

The ECM is estimated by ordinary least squares (OLS) method. The estimation process considers the Hendry ‘general-to-specific’ strategy (1992). General model is started by having drift term, three seasonal dummies, intervention dummies ($D_{08}, D_{011}, D_{af gw a}$), lag of error correction terms and lag length of eight for each first difference variables (exchange rate, prices, outputs, interest rates). The specific model is achieved by dropping the insignificant lags. The parsimonious ECM model with t-ratios in parentheses is as follows:

$$
\Delta r_t = (-2.77) + 0.22\Delta r_{t-7} - 0.23\Delta a_{t-7} - 0.01\Delta i_{t-7} - 0.03D_{af gw a} + 0.03EC1_{t-1} + 0.07EC2_{t-1} - 0.27EC3_{t-1} + 2.19\Delta p^f_{t-4} + 1.19\Delta p^f_{t-2} + 1.61\Delta p^f_{t-1} - 1.16\Delta y^f_{t-1} + 0.22\Delta y_{t-1} - 0.42\Delta y_{t-4}$$

$$
\text{Adj } R^2 = 0.40 \quad F_{(13,922)} = 9.93 \quad \text{prob (0.000)}
$$

The residual of parsimonious ECM satisfied the diagnostic tests of Breusch and Godfrey (1981) LM test of no serial correlation ($\chi^2_{(4)} = 4.28$), Engle’s (1982) no autocorrelation conditional heteroskedasticity (ARCH) LM test ($\chi^2_{(1)} = 1.40$ and $\chi^2_{(4)} = 3.56$) and Jarque-Bera normality test ($\chi^2_{(2)} = 5.47$ ) at 5 percent level of significance.
The estimated coefficients of ECM in Equation (17), show that in short run exchange rate immediately responds to change in foreign price level, domestic and foreign real output and domestic and foreign interest rates. The presence of lag of dependent variable makes the short run dynamic ECM as an autoregressive model. Its estimated coefficient indicates that a one percent depreciation in preceding seventh quarter (approximately two years back) results in the appreciation of current exchange rate by 0.22 percent.

In short-run change in foreign price level has dominant effect on the nominal exchange rate among the other variables, due to its higher coefficient. The positive sign of change in foreign price level indicates that increase in foreign price level immediately depreciates the domestic currency in the short run rather than appreciating it as suggested by the theory. It confirms the finding of Alam and Ahmed (2010) that Pakistan is a growth driven economy and increase in relative price of imports may not reduce the import demand. Pakistan’s major imports consist of petroleum products, essential capital goods and machinery goods. These goods contributed more than 50 percent share of total imports and among these goods Petroleum Group only constituted the largest share in our import bill that is 32 percent in 2010 (State Bank of Pakistan). An increase in oil prices disturbs balance of payment and puts downward pressure on exchange rate which makes imports more expensive [Malik (2008); Kiani (2010)].

A change in domestic output level in preceding quarter depreciates the domestic currency by 0.22 percent, but a change in four quarter previous output level results in the appreciation of currency by 0.42 percent. This result is consistent with Ahmed and Ali (1999) study, in which they suggest that a shock in output initially depreciates the domestic currency but after four periods it appreciates the domestic currency.

The estimated coefficients of lagged change in domestic and foreign interest rate are significant and negative. According to estimates, nominal exchange rate immediately appreciates due to change in domestic and foreign interest rates.

Among the intervention dummy variables only $D_{Afghan}$ is found to be significant in short run dynamic model. Its negative coefficient signifies the appreciation of rupee. During the period of US war against terror in Afghanistan the total US foreign assistance received by Pakistan since fiscal year 2002 is $20 billion. This is more than the aid Pakistan received from the US between 1947 and 2000, which is $12 billion [Epstein and Kronstadt (2012)].

The absence of financial crisis dummy variable does not imply that nominal exchange rate of Pakistan is independent of financial crisis. But the reason is the ignorance of financial sector in the Keynesian model. Therefore, the effect of financial crisis will be clearly measured in those models that incorporate the financial sector such as monetary and portfolio models of exchange rate.

Theoretically, sign of error correction term should be negative and significant. The negative sign confirms adjustment towards equilibrium state. In our analysis, coefficient of first error correction term is positive and statistically significant, while the coefficients of second and third error correction terms obey theoretical definition that is negative and significant.

The result of $EC_{1,t-1}$ and $EC_{2,t-1}$ indicates that exchange rate overshoots from long run equilibrium path by 10 percent. The third error correction term demonstrates that
long run deviation of nominal exchange rate from its equilibrium path is being corrected by 27 percent every quarter. Therefore, the net convergence of exchange rate towards its equilibrium state is 17 percent per quarter. The time required to remove 50 percent of disequilibrium from its exchange rate equilibrium path is three quarters (nine months).²

Finally, the stability of ECM’s parameters are examined by utilising Cumulative Sum of Recursive Residuals (CUSUM) and Cumulative Sum of Squares of Residuals (CUSUMSQ) test. The plots, provided in Figure 1 and Figure 2, show that CUSUM and CUSUMSQ remain within the 5 percent critical bound, suggesting that there is no significant structural instability and residual variance is stable during the analysis period.

![Fig. 1. Cumulative Sum of Recursive Residuals](image1)

![Fig. 2. Cumulative Sum of Squares of Recursive Residuals](image2)

²The time required to remove the $x$ percent of disequilibrium from its equilibrium path is determined as $(1 - EC)^t = (1 - x)$, where $t$ is required number of periods to dissipate $x$ percent of disequilibrium.
6. OUT OF SAMPLE FORECASTS

Mark (1995) and Cushman (2007) methodology of recursive regression has been adopted to generate multi-step-ahead forecast from Keynesian and random walk models. The methodology starts by dividing the data set, containing \( t=1, 2, \ldots, T \) number of observations that is from 1982:1 to 2010:2, into thirty seven subsamples \( t_1, t_2 \ldots, t_{37} \). The first subsample contains \( T-37 \) (smallest subsample) number of observations. We denote it by \( t_1 \) (ends at period 2001:1). The next subsample \( t_2 \) is extended by one observation; it contains \( T-36 \) number of observations (ends at period 2001:2), and so on the largest and last sample ends with \( T-1 \) number of observations, we denote it by \( t_{37} \) with ending period 2010:1.

The parsimonious error correction model Equation (17) is estimated for each subsample. This recursive procedure updates the estimated parameters in each subsample due to the inclusion of new data point. Each subsample estimated error correction model has been used to construct a one quarter ahead forecast to sixteen quarter ahead forecast. This results in 37 one quarter ahead forecast, 36 two quarter ahead forecast and so on till 22 sixteen quarter ahead forecast. Forecasted values are also obtained from random walk models for each subsample.

The forecasting performance of each forecast horizon under Keynesian exchange rate and random walk models are evaluated by using standard root mean squared error (RMSE) and Theil’s U statistics. Theil’s U statistics computes the ratio of the RMSE of the Keynesian model to the RMSE of random walk models. If this ratio is less than one then structural model on average provides better forecast than benchmark. Finally, statistical significance of each forecasting horizon is judged with the Diebold and Mariano (DM) (1995) test statistics.

Table 6 gives the result for RMSE of different models at 1, 4, 8, 12 and 16 forecasting horizons. It can be noted that RMSE of Keynesian model is smaller than the RMSE of benchmark random walk models, with and without drift, at all out of sample forecast horizons. Therefore, it is easy to conclude that Keynesian model yields better forecast for exchange rate than random walk models. Theil’s U coefficient at each forecasting horizon is reported in Table 7. This coefficient again supports the dominance of structural model over the random walk models at every horizon.

RMSE and Theil’s U factor do not provide any idea of the significance of the difference in the forecasting performance. Therefore, final conclusion will draw on DM test statistics. Table 8 lists the DM statistics and its associated probability values at various horizons, to significantly test whether the mean square error of one forecast is better than another.

First part of Table 8 takes random walk model without drift as benchmark model in loss differential function. The DM test statistics confirm that the predictive accuracy of Keynesian model is significantly more accurate than the random walk model at long forecast horizon i.e. \( k=12, \) and 16. The success of structural models at long horizons is consistent with Mark (1995) and Chinn and Meese (1992). Second part of Table 8

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3 Only the difference is in the construction of subsamples, Mark (1995) has considered forty subsamples and Cushman (2007) has followed Hansen and Juselius (1995) methodology and constructed thirty seven subsamples. This study has considered the later approach to elude the problem of smaller sample size at long horizon forecast and make the DM test statistics more reliable.
Table 6

Out-of-Sample Forecast Evaluation: RMSE

<table>
<thead>
<tr>
<th>RMSE</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>RW Model</td>
<td>0.048</td>
</tr>
<tr>
<td>RW with Drift Model</td>
<td>0.030</td>
</tr>
<tr>
<td>Keynesian Model</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Table 7

Out-of-Sample Forecast Evaluation: Theil’s U

<table>
<thead>
<tr>
<th>Model</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Benchmark: RW Model</td>
<td></td>
</tr>
<tr>
<td>Keynesian</td>
<td>0.793</td>
</tr>
<tr>
<td>Benchmark: RW with Drift Model</td>
<td></td>
</tr>
<tr>
<td>Keynesian</td>
<td>0.507</td>
</tr>
</tbody>
</table>

Table 8

Out-of-Sample Forecast Evaluation: DM Test Statistic

<table>
<thead>
<tr>
<th>Model</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Benchmark Loss Function: RW Model</td>
<td></td>
</tr>
<tr>
<td>Keynesian</td>
<td>0.573\a</td>
</tr>
<tr>
<td>(0.570)</td>
<td></td>
</tr>
<tr>
<td>Benchmark Loss Function: RW with Drift Model</td>
<td></td>
</tr>
<tr>
<td>Keynesian</td>
<td>1.133\a</td>
</tr>
<tr>
<td>(0.265)</td>
<td></td>
</tr>
</tbody>
</table>

\*Note: \** Represents the acceptance of null hypothesis of equal forecast.
A probability value of DM statistics is in brackets.

compares the difference in the forecasting performance of the structural models to the benchmark random walk with drift model. DM test statistics clearly states that parsimonious cointegrated Keynesian model easily beat the random walk model with drift at every horizon except the first. This finding confirms the remarks of Faust, et al. (2003) that it is easy to beat the random walk model with drift than the random walk model without drift.

7. CONCLUSION

This paper has empirically analysed the Keynesian exchange rate model by employing Johansen and Juselius (1988, 1992) cointegration method. Trace test has found three long run relationships among exchange rate, prices, interest rates and output levels. The symmetry restrictions on price coefficients and output coefficients and proportionality restriction on price coefficients are only satisfied by maximum likelihood
ratio test. This study has tested the various forms of PPP, UIP and their combinations to identify the cointegrating vectors. The results support the validity of PPP with the presence of other fundamental variables such as unrestricted output level and interest rates. However, UIP condition holds on its own. Based on these restrictions, further, the first cointegration vector has defined the current account, the second vector has explained the UIP and the last vector has described the Keynesian approach to exchange rate determination. The entire coefficients (except the interest rates) estimated in the system are significant and according to theory. The error correction terms suggest that the net convergence of exchange rate towards its equilibrium state is 17 percent per quarter and three quarters are required to remove 50 percent of exchange rate misalignment from equilibrium path.

The out of sample forecasting results suggest that in case of Pakistan Keynesian exchange rate model outperforms the random walk model, with and without drift, to accurately predict the nominal exchange rate. This finding is attributable to the parsimonious error correction model, which includes lag of dependent variable and fundamental variables to exchange rate determination, error correction terms and financial crisis dummy variable. Therefore, it captures the interruptions in the economy and explains the significant part of instability and outliers in exchange rate series.

The main policy implications drawn from this study are:

- The maintenance of PPP ensures that obtaining unlimited benefits from arbitrage in traded goods is not possible. Therefore, Pakistan is unlikely to improve its external competitiveness against U.S.
- Validity of PPP and UIP allows the use of inflation differentials and interest rate differentials to forecast long-run movements in exchange rates.
- The exchange rate of rupee against US dollar is significantly determined by output levels, prices and interest rates. Therefore, interaction between good and capital assets market is required to study exchange rate dynamics in Pakistan.

REFERENCES


Hina and Qayum


