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CONTENTS

ABSTRACT

We show that the rationality arguments used to establish the existence of subjective probabilities depend essentially on the identification of acting-as-if-you-believe and actually believing. We show that these two ideas, the pretense of knowledge about probabilities, and actual knowledge about probabilities, can easily be distinguished outside the restricted context of choice over special types of lotteries. When making choices over Savage-type lotteries, rational agents will act as if they know their subjective probabilities for uncertain events, but they will reveal their ignorance in other decision making contexts. This means that subjective probabilities cannot be assumed to exist, except when there is objective warrant for them.

JEL Classification: B40, C11

1. INTRODUCTION

"… *the race is not always to the swift, nor the battle to the strong* …" Ecclesiastes 9:11

The rapid rise and surprising fall of logical positivism is the most spectacular story of twentieth century philosophy. The positivists sought to replace scientific concepts referring to unobservables by their observable implications; for instance, unobservable gravity can be replaced by the observable elliptical orbits. In economics, unobservable preferences can be replaced by observable choices. This idea, that science was concerned primarily with observable phenomena, was extremely influential in shaping the approach to a wide variety of different areas of research. In particular, both the objective (frequentist) and the subjective (Bayesian) schools of thought were deeply influenced by positivist ideas, which required an observable definition for the unobservable probabilities. As a recent survey by Hands (2009) shows, even though logical positivism collapsed in mid twentieth century, the central ideas continue to be widely believed among economists. Our goal in this paper is show that the arguments for the existence of subjective probabilities are based on the central logical positivist strategy of replacing unobservable beliefs by observable choices over lotteries. Just as this strategy proved to be untenable in the context of science, so it fails in the context of human behaviour. Beliefs cannot be identified with choices over lotteries in the manner assumed by subjectivist arguments. Due to the continuing influence of positivist thought, this defective link in the argument has not clearly been identified in the literature.

We will replicate the subjectivist argument to show that every agent must act as if he has subjective probabilities in order to act rationally in certain decision-making environments. However, this "acting as if" does not mean that the agent has the beliefs that he appears to display. By switching to other decision-making contexts, we can reveal his lack of knowledge about the probabilities, which he appeared to know in a different context. It was this failure of the equation between unobservable scientific terms, and their observable implications, which eventually led to the collapse of logical positivism; see, for example, Suppe (2000) for details.

1.1. Intuitions About Risk and Uncertainty

A simple common sense and intuitive understanding of the world around us suggests that we do not know the probabilities of many random events which occur in our daily lives. Introspection does not reveal to me a number that I could state as my subjective belief about the probability of rain tomorrow. This intuition was formalised by Keynes (1921) and Knight (1921), who argued that situations where we lacked knowledge of probabilities (uncertainty) were fundamentally different from those where such knowledge was available (risk). A simple paradigmatic example is the difference between a horse race and a roulette wheel. Whereas there would be widespread agreement on probability calculations related to outcomes of a roulette wheel, we would be at a loss to find probability numbers to use for similar calculations relating to the outcomes of a horse race. While we do not expect to see any disagreements about roulette wheel probabilities, we could confidently predict substantial disagreements about horse race probabilities.

In a logical tour-de-force, Ramsey (1926) and De-Finetti (1937) and others showed that the difference between risk and uncertainty is an illusion. The key argument was that rational decision-making in situations of uncertainty requires acting as if you assign probabilities to uncertain events, and then doing the standard calculations made to evaluate risky events (with known probabilities). Furthermore, if you do not act in a way that is consistent with the existence of subjective probabilities, then you can be made to suffer certain losses—A Dutch Book can be made against you. This is inconsistent with rational behaviour, and hence it appears that rationality requires the replacement of uncertainty by risk. Savage (1954) played an important role in presenting a clear axiomatisation of rational behaviour which seems to require the existence of subjective probability. Anscombe and Aumann (1963) showed that despite the apparent difference between horse races and roulette, rational choices over lotteries related to both uncertain and risky events required the use of the similar subjective probabilities.

These formal arguments of the subjectivists won the battle against the strong intuition that there was a fundamental difference between uncertainty and risk, as advocated by Keynes and Knight. Even Keynes (1931) wrote “So far I yield to Ramsey—I think he is right,” apparently agreeing with the subjectivist formulation of probability. As a result, with minor exceptions, the concept of uncertainty virtually disappeared from the literature. More recently, strong empirical evidence for the presence and importance of uncertainty has led to re-emergence of interest in this concept from many different angles. In particular, Taleb (2007) has forcefully advocated the importance of “Black Swans,” events which are completely unpredictable from past experience. However, these challenges to the subjectivist view are based on observations about human behaviour and empirical experience, and do not address the logic of the subjectivist argument. In this paper, we show that the central argument for the existence of subjective probabilities is defective, and does not prove what it appears to prove.

The persuasive force of the arguments for subjective probabilities rests on two key ingredients. One is the positivist idea that since beliefs about probability are unobservable, they may be equated with their observable manifestations in terms of choices over lotteries. The second is the major difficulties which arise in defining and understanding single-case probabilities, which make it plausible to say that our intuitions about probability are simply a result of confused thinking. Our treatment addresses both of these difficulties, by showing clearly how to separate beliefs from actions-according-to-belief, and by proving a context where the difference between "knowing" a probability and not knowing it has a clear interpretation.

2. THE FUNDAMENTAL THEOREM OF SUBJECTIVE PROBABILITY

In what might be called *The Fundamental Theorem of Subjective Probability*, it is shown that rational choices over lotteries based on uncertain events require assigning subjective probabilities to random events. There are many variants of this basic theorem, originally developed by Ramsey (1926) and De-Finetti (1937). *Foundations of Statistics* by Savage (1959) is a convenient reference point with a clear exposition, references to the literature, and some historical background. This entire programme, of deriving subjective probabilities and utilities of payoffs from axioms of rational behaviour can be carried out in several different ways. In this section, we will derive a simple and transparent version of this Fundamental Theorem, which works without any rationality assumptions.

2.1. Setting Up Scales of Measurement

Let us consider how we could find out the subjective probability that rational agent Prabhavati assigns to an uncertain event **E** (like the chance of Black Beauty winning a horse race, or of rainfall tomorrow). A natural method is to first set up some scales of measurement. For this purpose, let U(n) be an urn containing 100 balls, such that n of these balls are black. Let **F(n)** denote the event that a ball chosen at random from the urn U(n) is black. Consider the lotteries *LF(n)* which pay $100 when the event **F(n)** occurs—that is, when a black ball is drawn from an urn with n black balls out of 100 total balls.

Hajek (2012) list seven different categories of theoretical interpretations of probability, while de Elía and Laprise (2005) document an equally wide variety of interpretations among forecasters and public in the extremely applied and practical domain of probabilistic weather forecasting. Despite this huge variation, we would expect to find unanimous agreement that the preferences of Prabhavati would be monotonic, so that *LF(0) < LF(1) < LF(2) … < LF(100)*; here the inequality denotes the preference ordering of Prabhavati. When there are more black balls in the urn, the chances of drawing a black ball, and winning the prize of $100 must be larger, almost regardless of how we think about probability. Without taking any particular stance on probability, we assume that Prabhavati's preferences over these 101 lotteries *LF(n)* for n=0,1,2,…,100 are strictly monotonic increasing in n. These lotteries set up a scale of measurement which we can use to evaluate any other probabilities, subjective or objective, that Prabhavati may or may not have. Note that issues of utility measurement are not involved—all that is required is that Prabhavati prefers $100 to $0.

2.2. Defining Subjective Probability

Next consider an uncertain event **G**—for instance, "it will rain tomorrow", or "horse X will win the race". We define the subjective probability p of **G** by comparing it to the reference probabilities of the risky events **F(n)**. Let *LG* be a lottery which pays $100 when the event **G** occurs, and $0 otherwise. We will define subjective probability in a natural way, by comparing how a rational agent Prabhavati chooses between *LG* and *LF(n)*.

DEFINITION: We say that agent Prabhavati has subjective probability p for event **E** if Prabhavati prefers *LF(n)* to *LG* if and only if n% is greater than p.

If Prabhavati has subjective probability p for event G, then we can determine this to within 1 percent by observing her choices between *LF(n)* and *LG*. We will always be able to find a unique integer n* such Prabhavati prefers *LF(n)* to *LG* for all n≥n* and she prefers *LG* to *LF(n)* for all n<n*. In this case the subjective probability of Prabhavati for the event **G** lies between (n*–1)% and n*%.

The key argument of the paper, made below, involves separating preference from choice behaviour. "Preference" is an unobservable internal condition of the heart, while choice is an observable decision. If Prabhavati has preferences as above, then her choices will reveal her preference. However, the reverse is not true. Prabhavati may make choices exactly as if she has subjective probability p near n*% without having any subjective probability in her heart. We now explain this further.

2.3. De-Linking Choices and Beliefs

By offering choices between the lottery *LG* and the lotteries *LF(n)* we can determine the subjective probability p of Prabhavati approximately, to within a percentage point. We are now in position to state our extremely elementary version of the fundamental theorem of subjective probability. The distinctive feature of this theorem, which differentiates it from similar theorems in the literature, is that it works without any assumptions at all—Prabhavati may be rational or irrational, knowledgable about event E or otherwise, and may make choices arbitrarily and thoughtlessly, or carefully and thoughtfully. In all cases, she will end up revealing her subjective probability (to within 1 percent) about the event E in a sequence of seven choices.

The Seven Steps Theorem: By offering Prabhavati a sequence of seven choices of between the lottery *LG* and the lotteries *LF(n)* for seven sequentially chosen values of n, we can learn the subjective probability p that Prabhavati assigns to the event **G**, with a maximum error less than 1 percent.

Proof: Start by setting n=50, and offer Prabhavati a choice between *LG* and *LF(n=50*). A choice of *LG* reveals p≥50 percent, while the choice of *LF(50)* reveals p≤50 percent (where p is the subjective probability of **G** for Prabhavati). Thus, from the first choice, we can learn whether $p \in [0{,}50\%]$ or $p \in [50\%, 1]$ We continue this process by splitting the interval within which p must lie, into two and offering Prabhavati a choice between the (approximate) midpoint lottery LF(n) and the lottery LG. At each stage, the range of possible values for the subjective probability *p* is halved. After 7 steps, the range of possible values for the subjective probability will be confined to some interval of the form [(n–1) percent, n%] as asserted by the theorem.

What is interesting about this theorem is that *every* sequence of choices reveals a subjective probability. All versions of this theorem available in the literature assume some properties of rationality, embodied in the form of coherent choices, on part the agent. In contrast, our theorem shows that every agent, regardless of whether he is rational or irrational, and regardless of his state of knowledge or ignorance about the event G, will be forced to reveal a fairly precise value p of his subjective probability for G. This clarifies a hidden structure of these Fundamental Theorems. The existence of subjective probability emerges directly from equating choices with states of belief; it does not require any other assumptions of rationality or coherence. As we will clarify further, the set of possible beliefs is very large, while the set of choices is very small, and the mapping from beliefs to choices is many-to-one. This means that from beliefs we can infer choices, but the mapping cannot be inverted. The existence of subjective probabilities depends on making this impossible inversion. The central argument is that agents with belief B will make choices C. If the agent makes choices C, then he has beliefs B. This argument is only valid if the mapping is one-to-one.

We will now discuss the role played by rationality, coherence, and knowledge of the agents, in determining their choices over lotteries based on the event G. The theorem above shows that these factors do not matter for establishing the existence of subjective probabilities.

3. DISTINGUISHING KNOWLEDGE FROM IGNORANCE

A serious problem in resolving puzzles created by personal probabilities has been the difficulty of defining and/or understanding what it means to "know" the probability p of a single event G. The problem of defining "single-case" probabilities is discussed/reviewed in Hajek (2012). In the context of the problem under study, we can create a model which clearly differentiates between a large variety of states on knowledge about probabilities.

3.1. A Model for Uncertainty and Risk

We have 99 Urns, U(1), U(2), …, U(99), each containing the indicated number of black balls within a total of 100 balls. The event **F(n)** occurs when a random draw from Urn U(n) results in a black ball. The lottery *LF(n)* pays $100 when event **F(n)** occurs. We can model the difference between uncertainty and risk by using the events **F(n)** as follows. Suppose that experimenter Exposito is trying to learn about human behaviour in face of uncertainty and risk. Exposito selects a particular fixed value N between 1 and 99, and draws a ball at random from Urn U(N). We label this two step procedure—Exposito's choice of N and drawing of a black ball from urn U(N)—as the event **G**. In order to model risk and uncertainty, Exposito reveals the value of the integer N to subject Kanza, but conceals it from subject Ignatius. In this situation, the event **G**=**F(N)** has known probabilities for Knowledgable Kanza, but unknown probabilities for Ignorant Ignatius. Thus, the event **G** is risky, like roulette for Kanza, but uncertain, like a horse race, for Ignatius. If the argument for the existence of subjective probabilities is correct, then we should not be able to distinguish between the behaviours of Kanza and Ignatius.

Intuitively, it seems very clear that knowledge of N puts Kanza in a very strong position relative to Ignatius when it comes to choices over lotteries. Indeed, it seems hard to see how anyone could argue that uncertainty and risk are the same—how can it be that knowledge of N does not matter when it comes to choice of lotteries from the Urn U(N)? We now reconstruct the subjectivist argument that leads to this tempting but false conclusion. The first part of the argument has already been presented. By offering Kanza and Ignatius seven choices of lotteries based on **G**=**F(N)** and those based on **F(n)** for particular fixed known values of n, we can force them to "reveal" their subjective probabilities for the event G. It is immediately obvious that Kanza will always choose *LF(n)* if n>N, and *LG* if n<N, and end up revealing p close to N%. But how will Ignatius behave?

3.2. The Ellsberg Paradox

Following the Seven Steps Theorem, as a first step Exposito offers Ignatius a choice between the lottery LF(n=50) with known success probability 50 percent, and LF(n=N) with unknown success probability N%. Due to his ignorance of N, Ignatius can only make the choice arbitrarily, choosing one or the other according to his feelings about how Exposito may have chosen N. He does not have a "preference"; unlike Kanza, he does not know which of the two choices is better. If he chooses LF(50) with known probabilities, this does not reveal that he "knows" that N<50. It is this confusion between preference and choice which creates the Ellsberg paradox. The crucial point is that there are only two choices, but there are many possible states of knowledge. Suppose K1 is knowledge that N>50 and K2 is knowledge the N<50 and K3 is knowledge that N is between 40 and 60. All three states of knowledge map into the same two choices. K1 clearly maps to the choice of *LF(N)*, while K2 maps to the choice of *LF(50)*, but K3 must be mapped to one of the same two choices. When we try to invert the map, to learn about knowledge from beliefs, we cannot do so. Nonetheless, the subjectivist argument is based on this inversion. The choice between LF(N) and LF(50) reveals that either N>50 or N<50, so we argue that the subject must know which of these inequalities holds. This is precisely the basis of the Ellsberg paradox.

External observer Ellsberg is watching this experiment, but he does not know about the state of knowledge of Kanza and Ignatius—these are unobservables to him. Ellsberg observes that at the first step, Ignatius expresses a distinct preference for LF(50) over LG=LF(N). This reveals to Ellsberg that Ignatius (is acting as if he) knows that N < 50. To test this, Ellsberg constructs the lottery LG* which pays $100 when a black ball is NOT drawn from urn F(N); G* is the complement of the event G and the sum of their probabilities is necessarily one. Next Ellsberg offers Ignatius a choice between LG* and LF(50). Much to his surprise, Ignatius again expresses a distinct preference for LF(50), revealing that N>50. These two revelations conflict with each other, creating the Ellsberg paradox. The Ellsberg paradox disappears if we do not equate choices with preferences. The choice of *LF(50)* does not reveal that Ignatius has knowledge that N≤50—he is making an arbitrary choice because the experimenter forced him to do so. Note that the revealed preference reasoning would be perfectly valid for Kanza—her choices will indeed reveal the value of N which she knows to the observer.

There is strong empirical evidence that most (but not all) people prefer risk to uncertainty, and hence choose lotteries with known probabilities over those with unknown probabilities. This has been labelled "ambiguity aversion" and there is a large literature, both theoretical and empirical, on this topic. This literature takes the Ellsberg phenomenon as a given, and constructs theoretical models for human behaviour as well as empirical measures of the strength of the effect. Our goal in this paper is to go beyond an empirical demonstration of the failure of subjectivist argument, and to explain why the logic of the argument is wrong.

3.3. Coherent Extension

So far, we have not presented the full strength of the fundamental theorem, which invokes rationality and coherence. This is in order to isolate to role of these assumptions about human behaviour. As we have seen, contrary to what is widely believed, the existence of subjective probabilities comes directly from identifying choices with beliefs, without any rationality assumptions: If Ignatius chooses LF(N) over LF(50) then he must believe that $N \geq 50$, so that the probability of event LF(N) is greater than 50 percent. In fact, Ignatius does not know N, and so his choice cannot reveal this knowledge, which he does not have.

Surprisingly, the game is not over. The subjectivists have a powerful argument at their disposal, which appears to overcome the strong objections that have been made. Except for a few voices who have argued in vain against it, this argument has managed to convince the majority, and dominates in the literature. This argument goes as follows. Let us concede that Ignatius has made arbitrary choices in the first seven steps, revealing a probability p for the event G which he does not necessarily believe in. Nonetheless, now that he has revealed this probability, he is compelled to act in accordance with this revealed belief; failure to do so would be irrational. We will now prove this, to complete our proof of the fundamental theorem. First, we need some preliminary definitions.

Well-Funded experimenter Exposito plans to offer subject Rational Robert a sequence of 101 choices, asking him to choose between LG and LF(n) for n=0,1,2,…,100. Let us say that a sequence of choices is monotonic if all the choices are *LG* up to a certain value of n=n*, and switch to *LF(n)* for all n>n*. It is obvious that if Robert knows the probability p of the event G, then his choices will be monotonic, switching from LG to LF(n) when n% exceeds p. Thus, every monotonic sequence "reveals" Robert's subjective probability p for the event G to be in the interval [n*%, (n*+1)%), if Robert switches to choosing LF(n) over LG for n>n*.

Untrained intuition suggests that if Robert does not know the value of p, his decisions may fail to be monotonic. He might set some arbitrary value of p and change this, in accordance with his feelings at the time of decision. However, subjectivist arguments show that rationality requires monotonic choices, consistent with the existence of a subjective probability p.

The 101 Choices Theorem: Every rational agent must make monotonic choices in the sequence of 101 choices described above, thereby revealing his subjective probability for the event G to within 1 percent.

Proof: Unlike the seven steps theorem, where incoherent or irrational choices are not possible by construction, 101 choices give agents a chance to express their lack of knowledge of p. If a sequence of choices is not monotonic, then it does not map to any subjective probability, and therefore expresses lack of knowledge. However, it is irrational to make non-monotonic choices, as is easily shown. If choices are not monotonic, then for some integers m,n such that m<n, Robert chooses *LF(m)* over *LG*, and *LG* over *LF(n)*, thereby obtaining the lotteries *LF(m)* in the first choice and *LG* in the second choice. Consider swapping these two choices in order to make the sequence monotonic. Making the swap, Robert will get LG in the first choice and LF(n) in the second one.

This second set of lotteries dominates the first one, since LF(n) is preferred to LF(m) because n is greater than m. The lottery LG is the same in both outcomes. This proves that rational choices must be monotonic, and hence reveal some subjective probability.

This theorem can be substantially extended. We have demonstrated the rational choice is coherent over 101 basic lotteries. By adding plausible rationality axioms, we can extend this to an elaborate and complex structure of decisions regarding the uncertain event G. All such decisions must be coherent with the choice of a subjective probability p for G. The rational necessity for coherent decision making over a large class of lotteries about G creates the impression that *all* decisions about G must be coherent with a subjective probability p for G. This is exactly what the subjectivists have forcefully argued. It does not matter if our initial choices, feeling out our personal intuitions about p, are arbitrary. Rational decision making requires us to pick a subjective probability p, and then to stick to it for all subsequent decisions about G. In this case, everyone acts as if he has a subjective probability, and there is no difference between those who actually have subjective probabilities, and those who merely pretend to do so for the sake of consistency and rationality. This means that, contrary to Keynes and Knight, there is no essential difference between risk and uncertainty.

This argument fails because there are other types of decisions, not considered by the subjectivists, which clearly reveal the difference between ignorance and knowledge. Furthermore, there are many more manifestations of lack of knowledge, even within the confined and restricted set of situations considered by the subjectivists. After the seven steps, both Kanza and Ignatius will conform to the probability they revealed, when they are later offered the 101 choices. So even though Ignatius made arbitrary choices, in the process of making these choices he unknowingly committed himself to a particular probability p for G, which corresponds to a particular guess at the N chosen by experimenter Exposito for the urn U(N). An important consequence of this, that choices create preferences, is highlighted in Ariely and Norton (2008). More explicitly, while the knowledge of Kanza about probabilities governs her choices over lotteries, the reverse is true for Ignatius. He makes choices, which lead him to a commitment to a subjective probability, which is preserved for later choices.

4. CONSEQUENCES AND EXTENSIONS

According to the fundamental theorem, we can make every agent reveal a subjective probability for any event G, whether risky or uncertain, in seven steps. Furthermore, subsequent choices over certain Savage-type lotteries must be coherent with this initially revealed probability in order to be rational. This is an exact description of the phenomenon of “Coherent Arbitrariness” discovered empirically by Ariely, *et al.* (2003), as we now discuss.

4.1. Coherent Arbitrariness

Our theoretical arguments given here are strongly supported empirically by Ariely, *et al.* (2003). They show that in situations where agents do not know their preferences over objects, they make arbitrary decisions at an early stage in a sequence of choices, and make later decisions to cohere with their early decisions. As a result, to an outside observer it appears as if the agents are acting coherently, when in fact they are acting arbitrarily subject to a rationality constraint. Ariely, *et al.* (2003) call this phenomenon "Coherent Arbitrariness".

As we have shown, rationality constrained Ignorant Ignatius will appear to behave as if he has a subjective probability p, just like Kanza. However the "coherently arbitrary" behaviour of Ignatius can be differentiated from that of Knowledgeable Kanza in many ways. We just have to step outside the circumscribed framework of Savage lotteries. Experimenter Exposito has chosen urn U(N) and revealed N to Kanza, while concealing it from Ignatius. In the Savage lotteries, all knowledgeable agents will make identical choices, while all ignorant agents will make arbitrary choices and exhibit diversity of opinions about N, without being able to accurately hit the correct value chosen by the experimenter. Furthermore, we can create a meta-lottery, where Exposito offers a prize of $1000 dollars for a correct guess of the value N he has chosen—which is exactly the subjective probability of the event G. If the cost of entering the competition for this prize is $100, Kanza will happily enter, while Ignatius will decline. In general, the particular structure of Savage lotteries compels Ignatius to pick a probability and stick to it, but it does not give him knowledge of p. This is contrary to the standard understanding of Bayesians, who think that making such choices brings out hidden knowledge inside the heart about the probability.

4.2. Objective Probability

De Finetti's (1974) treatise on the theory of probability begins with the provocative statement PROBABILITY DOES NOT EXIST; meaning that objective probability does not exist. He goes on to argue that subjective probabilities do exist, and can be defined by the means of subjective expectations over outcomes—previsions, in his language. Savage (1954) translated De Finetti's arguments into the language of choices over lotteries, and recreated his argument which is an elaborate version of our fundamental theorem of subjective probability. We have shown that the argument is valid, but does not imply what it is taken to imply. That is, within the framework of choices over Savage lotteries, rational agents will indeed behave coherently and appear to act as if they have subjective probabilities, displaying "coherent arbitrariness". However, outside this framework, in many other types of choices and decisions, they will reveal their ignorance of the probabilities in question. Thus, the fundamental theorem does not establish the existence of subjective probabilities.

Furthermore, our framework provides a strong argument for the existence of objective probabilities. Consider the choice between the lotteries *LF(m)* and *LF(n)* which pay $100 when a black ball is drawn from the urns U(m) and U(n) respectively. If integers m and n are known to the agents making the choice, all rational agents will choose the lottery from the urn with the higher integer, having more black balls. It is clear that this is a feature of the urn and ball setup, and not a feature of subjective beliefs of the agents. Because this is invariant across agents, this is an objective probability. Furthermore, it is obvious that this is a single case probability, which has nothing to do with long run frequencies of occurrence. Our framework supports the "propensity" interpretation of probability, according to which urns with more black balls have a greater propensity to have black balls chosen from them. This is a physical and objective feature of the setup by which the balls are drawn from the urns.

5. CONCLUSIONS

Our main argument can be summarised very simply. Consider the event **F(n)** which occurs when we randomly pick a black ball out of an urn U(n) containing 100 balls among which n are black. If we know n, this event has known probability n/100. Consider however, a rational agent who does not know n. Suppose that he is offered a choice between lottery *LF(n)* and *LF(50),* which pay $100 when a black ball is drawn from U(n) and U(50) respectively. The agent who does not know the value of n cannot make a rational choice—he lacks the information necessary to do so. He will choose arbitrarily. If he chooses *LF(n),* this does not reveal that he believes $n>50$ and if he chooses *LF(50),* he does not reveal that $n<50$. This confusion between choices and beliefs lies at the heart of the arguments for subjective probability. It leads to the false conclusion that even though rational agents do not know the value of n, they must know whether $n\geq 50$ or $n\leq 50$ – their choice will reveal one of these two beliefs.

The theory of subjective probability, launched by Ramsey, De-Finetti and Savage, among others, has come to play a significant role in many areas of social science. Starting from the position of an embattled minority, it has gained many passionate proponents, and acquired legitimacy and respect in many disciplines. Kyburg (1978) has described the many virtues of the theory which have led this remarkable performance. Nonetheless, he says that "I shall argue that although the theory appears to be all things to all people, in fact it is a snare and a delusion and is either vacuous and without systematic usefulness, or is simply false." In this paper, we provide some new simple and direct arguments which support these views of Kyburg, and show that subjective probability is indeed a snare and a delusion.

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