function for expenditure given by Equation (11) contains sufficient information to study the anticipated changes in wage rate.[1]

Finally, substituting the upper stage solution given by Equation (11) into the lower stage solution given by Equations (6) and (7) gives the λ-constant demand functions for consumption and leisure:

$$C(t) = Z\ [\ 1 + \{\ W(t)\}^{1-\theta}\ \alpha^{\theta}\ ]^{\frac{\theta-\sigma}{1-\theta}} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$E\text{–}H(t) = Z\ \{\ W(t)\ \}^{-\theta}\ \alpha^{\theta}\ [\ 1 + \{\ W(t)\}^{1-\theta}\ \alpha^{\theta}\ ]^{\frac{\theta-\sigma}{1-\theta}} \quad \ldots \quad (13)$$

These demand functions are used to find the relationship between the anticipated changes in income and consumption in the next section.

IV. CHANGES IN INCOME AND CONSUMPTION OVER TIME

According to Equation (13) work hours vary over the life cycle only due to anticipated changes in the wage rate. Therefore, if the age path of the wage rate is fully anticipated, the age path of income $Y(t) = W(t)\ H(t)$ is also known at the beginning of the horizon. The anticipated changes in income over the life cycle are given by the following age derivative, derived in Appendix C.

$$\dot{Y}(t) = [\ H(t) + Z\ \{\ W(t)\}^{-\theta}\ \alpha^{\theta}\ \{\ \theta + \sigma\ \{\ W(t)\}^{1-\theta}\ \alpha^{\theta}\ \}$$

$$B(t)\ ]\ \dot{W}(t) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where,

$$B(t) = [\ 1 + \{\ W(t)\}^{1-\theta}\ \alpha^{\theta}\ ]^{\frac{\theta-\sigma}{1-\theta}} - 1 > 0 \quad \ldots \quad \ldots \quad (15)$$

Equation (14) shows that income varies over the life cycle in the same direction as the anticipated wage rate.

Next, the planned changes in consumption can be found by taking the age derivative of Equation (12). The result is obvious:

$$\dot{C}(t) = (\theta-\sigma)\ Z\ \{\ W(t)\}^{-\theta}\ \alpha^{\theta}\ B(t)\ \dot{W}(t) \quad \ldots \quad .. \quad (16)$$

[1]The term "λ-constant" is due to MaCurdy (1981). He derived the λ-constant demand functions for consumption and leisure to study the response of consumption and work hours to anticipated changes in wage rate over the life-cycle.

To find the marginal consumption rate out of anticipated income, we divide the rate of change in planned consumption by the rate of change in anticipated income. The result is:

$$\frac{dC/dt}{dY/dt} = \frac{(\theta-\sigma)\, Z\,\{W(t)\}^{-\theta}\,\alpha^{\theta}\, B(t)}{H(t) + Z\,\{W(t)\}^{-\theta}\,\alpha^{\theta}\,[\,\theta + \sigma\,\{W(t)\}^{1-\theta}\,\alpha^{\theta}\,]\,B(t)} \quad \cdots (17)$$

From this Equation we can derive two important results.

Result 1: The marginal consumption response to anticipated income changes is positive (negative) if and only if the intratemporal substitution elasticity θ is greater (less) than the intertemporal substitution elasticity σ. If the two parameters are equal, the marginal consumption rate would be zero.

This result is explained as follows. An anticipated increase in the wage rate at age t affects consumption on two accounts. First, it makes consumption cheaper relative to leisure at age t. As a result the individual would consume more at age t. The magnitude of this effect depends on the size of the intratemporal elasticity of substitution between consumption and leisure, θ. Second, the anticipated increase in the wage rate at age t makes expenditure (on consumption and leisure) relatively more expensive at that age in comparison to other ages. Therefore, the individual plans to consume less at age t. The magnitude of this effect depends on the size of the intertemporal substitution elasticity of expenditure (or consumption) at different ages, σ. Since income varies directly with the wage rate, the sign of the marginal consumption response to in anticipated income follows the pattern described above.

Result 2: The marginal consumption rate out of anticipated income is less than one irrespective of the size of the two substitution elasticities.

Thus, while the individual may increase consumption in response to an anticipated increase in the wage rate under certain conditions on the substitution elasticities, he will increase savings under more general conditions.

V. CONCLUSIONS

A life-cycle model of consumption and work hours is studied. Using the class of utility functions with constant intertemporal and intratemporal elasticities of substitution, changes in labour income and consumption over the life cycle are derived. It is shown that the marginal propensity to consume out of anticipated labour income is positive if and only if the (intratemporal) elasticity of substitution between consumption and leisure at a given age is greater than the (intertemporal) elasticity of substitution between expenditure on consumption and leisure across two different

ages. In addition, it is shown that the marginal propensity to save out of anticipated labour income is positive under quite general conditions. This supports the idea that the Keynesian Absolute Income Hypothesis of the consumption function can be supported even in the life-cycle context.

Appendix

A. THE DEMAND FUNCTIONS AT THE LOWER STAGE

The maximizing conditions for the Lagrange problem Equation (5) are:

$$[\{C(t)\}^{1-1/\theta} + \alpha\{E-H(t)\}^{1-1/\theta}]^{\frac{\sigma-\theta}{\sigma(\theta-1)}} \{C(t)\}^{-1/\theta} = \lambda(t)$$

$$[\{C(t)\}^{1-1/\theta} + \alpha\{E-H(t)\}^{1-1/\theta}]^{\frac{\sigma-\theta}{\sigma(\theta-1)}} \alpha\{E-H(t)\}^{-1/\theta}$$

$$= \lambda(t)W(t)M(t) - C(t) - W(t)\{E-H(t)\} = 0 \quad \ldots \quad \ldots \quad \text{(A1)}$$

We can eliminate $\lambda(t)$ by taking ratio on the two sides of the first two equations. The result after simplification is:

$$C(t) = \{W(t)/\alpha\}^{\theta}\{E-H(t)\} \quad \ldots \quad \ldots \quad \ldots \quad \text{(A2)}$$

Solving Equations (A1) and (A2) simultaneously gives the lower stage solution in terms of the conditional demand functions for consumption and leisure given by Equations (6) and (7) respectively.

B. THE INDIRECT UTILITY FUNCTION

To find the indirect utility function we substitute the consumption and leisure demand functions ((6) and (7)) into the utility function (1) as follows:

$$U(t) = \frac{1}{1-1/\sigma}\left[\left\{\frac{M(t)}{1+\{W(t)\}^{1-\theta}\alpha^{\theta}}\right\}^{1-1/\theta} + \alpha\left\{\frac{\{(Wt)\}^{-\theta}\alpha^{\theta}M(t)}{1+\{W(t)\}^{1-\theta}\alpha^{\theta}}\right\}^{1-1/\theta}\right]^{\frac{1-1/\sigma}{1-1/\theta}}$$

$$= \frac{1}{1-1/\sigma}\left[\left\{\frac{M(t)}{1+\{W(t)\}^{1-\theta}\alpha^{\theta}}\right\}^{1-1/\theta}\left\{1+\{W(t)\}^{1-\theta}\alpha^{\theta}\right\}\right]^{\frac{1-1/\sigma}{1-1/\theta}}$$

Simplifying further, we obtain the indirect utility function (8).

C. THE AGE PATHS OF WORK HOURS AND INCOME

Differentiate Equation (13) with respect to t, we find the age path of work hours:

$$\dot{H}(t) = \theta Z\{W(t)\}^{-\theta-1}\alpha^{\theta}[1+\{W(t)\}^{1-\theta}\alpha^{\theta}]^{\frac{\theta-\sigma}{1-\theta}}\dot{W}(t)$$

$$-(\theta-\sigma)Z\{W(t)\}^{-2\theta}\alpha^{2\theta}[1+\{W(t)\}^{1-\theta}\alpha^{\theta}]^{\frac{\theta-\sigma}{1-\theta}-1}\dot{W}(t)$$

Collecting the common terms on the right hand side, we have

$$\dot{H}(t) = Z\{W(t)\}^{-\theta-1}\alpha^{\theta}[\theta+\theta\{W(t)\}^{1-\theta}\alpha^{\theta}-(\theta-\sigma)\{W(t)\}^{1-\theta}\alpha^{\theta}]$$

$$[1+\{W(t)\}^{1-\theta}\alpha^{\theta}]^{\frac{\theta-\sigma}{1-\theta}-1}\dot{W}(t)$$

$$= Z\{W(t)\}^{-\theta-1}\alpha^{\theta}[\theta+\sigma\{W(t)\}^{1-\theta}\alpha^{\theta}B(t)]\dot{W}(t) \quad \ldots \quad (A4)$$

where, $B(t)$ is given by Equation (15).

Next, the age derivative of income $Y(t) = W(t)H(t)$ is $\dot{Y}(t) = H(t)\dot{W}(t)+W(t)\dot{H}(t)$ which can be inferred from (A4). The result is given by Equation (14).

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Comments on "The Relationship between Income and Consumption in Life Cycle Models"

Let me start by saying that the paper by Dr Ahmad is an interesting and useful contribution to the debate on the relationship between income and consumption. The recent interest over this relationship is partly due to the paper by Lester Thurow in an 1969 issue of the American Economic Review in which he presented empirical evidence supporting a positive relationship between income and consumption. Thus, contradicting the life-cycle consumption theory which predicts no necessary relationship between consumption and income at any age. Lester Thurow explained this positive relationship in terms of credit market restrictions by arguing that credict market restrictions prevent consumers from borrowing as much against their future income as they desire at the going interest rate. Since income tends to increase with age and discounted future income cannot be fully transferred at the borrowing rate, this leads to an increase in the consumer's net worth which causes consumption to increase with age. Nagatani (1972) has explained the same phenomenon in terms of uncertainty of future income by arguing that a typical consumer buys less than he would in a riskless environment with the same expected income. However, consumption plans are successfully revised once expected income is realized. Subsequently, Heckman (1974) presented an alternative neo-classical model to explain Thurow's result. Rather than resorting to credit market restrictions or uncertainty, Heckman treats earnings as resulting from a life-cycle labour supply decision, where individuals are free to set their hour of work and wage rate change systematically over the life cycle. Heckman demonstrates that in such an environment the consumption path of market goods depend on the wage rate at each age. Heckman in his paper considered a very general form of the utility function to show the positive relation between income and consumption. Dr Ahmad in his paper has considered a very specific form, namely the constant elasticity of substitution, to derive the necessary and sufficient condition for a positive relation between income and consumption. I would imagine that it should be possible to obtain similar conditions by considering different forms of the utility function.

Let me now turn to some specific comments on the paper.

Although the type of model employed in the paper has been widely used in many other studies, the specific form adopted in this paper is highly simplistic. For

instance, both the interest rate and the rate of time preference are assumed to be zero. Similarly, initial wealth has been excluded from the analysis. However, because of the model being simplistic the results holds under some very special circumstances. For instance, within the life-cycle framework, the positive relationship between income and consumption holds only if the interest rate equals the rate of time preference, i.e., as long as the consumers are at a steady state equilibria. It has been shown elsewhere that if the interest rate and the rate of time preference differ, the association between income and consumption is not precise. Since the author has used a specific form of the utility function it would be interesting to know whether in a more general case the positive relation between income and consumption is possible over certain ranges of values of the parameters.

While the condition for positive relation between income and consumption, derived in this paper is important in its own right, it would be interesting to know under what sort of circumstances this is likely to hold and what policy implications are likely to emerge.

Finally, it has also been shown in the paper that under quite general circumstances, planned savings and anticipated labour income are positively correlated. With both interest rate and the rate of time preference assumed to be zero, it is not clear that in the model what motivates the consumers to save.

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