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Dynamic Properties of an Aggregate Econometric Model of Pakistan's Economy*

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I. INTRODUCTION

The use of econometric models for policy planning and decision-making is wide-spread in many developed as well as developing countries. One of the most vexing problems of such an exercise is to construct a model that could adequately reproduce the dynamic behaviour of an economy. The recent experience in modelling has shown that policy objectives could be achieved only by recognising the complex relationship between real and monetary variables. Such an integrated framework could be used not only to compute impact and dynamic multipliers and to determine the stability of the model, but also to evaluate the relative importance of fiscal and monetary policies.

In the present paper, this objective is achieved by constructing a linear yet dynamic macro-econometric model of Pakistan's economy.[1] This model although has a Keynesian structure, but it could easily and meaningfully be *solved* to determine the values of endogenous variables especially income in terms of pure exogenous variables. In order to establish the dynamic stability of the model, we seek to present the "necessary conditions" that will depend not only on the structure of the model, but also on the estimated paramters of structural equations. After establishing the stability of the model, the next step is policy evaluation. In this regard the impact and the dynamic multipliers will be computed. These multipliers will then be used to assess the relative importance of fiscal and monetary policy variables on income and other dependent variables such as consumption and investment. The time period under consideration ranges between 1959-60 and 1987-88 which includes dramatic events like two wars with India, nationalisation, the oil price hike, recession and floods.

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[1]Even though medium to large size econometric models exist for Pakistan, such as Naqvi *et al.* (1983); Naqvi and Ahmed (1986) and Naqvi, Khan and Ahmed *et al.* (1992) the objectives of the present paper would be achieved by specifying a relatively small econometric model.

The paper is planned as follows. In Section II the model is specified. Section III explains the methodology for dynamic analysis. Results of estimation and the impact and the dynamic multipliers are reported in Section IV and the final section contains the summary and conclusions.

II. SPECIFICATION OF THE MODEL

We specify a model within a Keynesian framework which includes five behavioural equations for private consumption, private investment, direct taxes, imports, and money demand and three income-related identities. The specification of the behavioural equations is as follows.

Consumption Function

Private consumption is hypothesised to depend on disposable income net of remittances (Y_t^{dn}), remittances (R_t^m), and the lagged value of consumption (C_{t-1}). The inclusion of the previous value of consumption is equivalent to a lag distribution of disposable income with geometrically declining weights. This specification is also consistent with stochastic implications of life cycle-permanent income hypothesis [Hall (1978)]. Symbolically, this relationship can be stated as:

$$C_t = \alpha_0 + \alpha_1 Y_t^{dn} + \alpha_2 R_t^m + \alpha_3 C_{t-1} + u_C$$

where u_C is the random disturbance term which is assumed to have zero mean and finite variance.

Tax Function

Disposable income is defined as income net of direct taxes. Since income and corporate taxes are levied mainly on non-agricultural income (Y_t^{na}), the tax equation specified below includes this variable as the only determining factor[2]: That is

$$T_t = \tau_0 + \tau_1 Y_t^{na} + u_T$$

where u_T is the stochastic disturbance which has similar properties as u_c.

Investment Function

An investment function with accelerator has previously been estimated for the large-scale manufacturing sector of Pakistan.[3] Investment, in the present paper, depends on the following list of explanatory variables. Change in aggregate income

[2]This simple specification for tax function makes no account for discretionary tax changes.
[3]See Naqvi *et al.* (1983).

(ΔY_t), interest rate (r_t) and lagged investment (I_{t-1}). The inclusion of lagged investment is justified on the grounds that actual investment never adjusts to the desired level instantaneously. Thus, the investment function is specified as:

$$I_t = \beta_0 + \beta_1 \Delta Y_t + \beta_2 r_t + \beta_3 I_{t-1} + u_I$$

Imports Function

Similar to the tax function, we have used a simple specification for imports which includes aggregate income (Y_t) and lagged values of imports (M_{t-1}) as the right-hand side variables. The model also includes a disturbance term u_M; i.e.,

$$M_t = \gamma_0 + \gamma_1 Y_t + \gamma_2 M_{t-1} + u_M$$

Money Demand Function

Several earlier studies have confirmed that the standard specification of the money demand function holds for Pakistan also.[4] The specification in the following is similar to the one used in Ahmed and Rafiq (1987) where demand for real balances depends on current income (Y_t) used here as a scale variable and interest rate (r_t). To incorporate the fact that the adjustment of portfolios to the equilibrium value does not occur in a single period, an adjustment mechanism is used. The resulting specification of the money demand function is given below:

$$m_t^d = \theta_0 + \theta_1 Y_t + \theta_2 r_t + \theta_3 m_{t-1}^d + u_m$$

The Identities of the Model

Along with the above stochastic behavioural equations, the model also includes the following definitional relationships.

(i) Disposable income $Y_t^d = Y_t - T_t$

(ii) Disposable income net of remittances $Y_t^{dn} = Y_t^d - R_t^m$

(iii) Non-agriculture income $Y_t^{na} = Y_t - Y_t^A$

where Y_t^A denotes agriculture income.

[4]See among others Mangla (1979); Khan (1980); Ahmed and Rafiq (1987) and Cornelisse and Martens (1989).

III. METHODOLOGY FOR DYNAMIC ANALYSIS

(a) Reduced Form Model

Using matrix notations, the structural form model will be:

$$B(L)\,Y_t + C(L)\,Z_t = u_t \qquad \ldots \qquad \ldots \qquad (3.1)$$

where *m* endogenous variables and hence *m* equations are represented by Y_t. Z_t are *k* exogenous variables and u_t is a vector of error terms. The lag operator defined by $LW_t = W_{t-1}$ is used with the following explanation:

$$B(L) = B_0 + B_1 L + \ldots + B_g L^g$$

$$C(L) = C_0 + C_1 L + \ldots + C_r L^r$$

Here B_0 is subject to normalisation. Expression (3.1) can now be used to derive the reduced form model that expresses Y_t (and other endogenous variables) in terms of predetermined variables as:

$$Y_t = B_0^{-1}\,[B_1 Y_{t-1} + \cdots + B_g Y_{t-g}] + [-(B_0)^{-1} C(L)\,Z_t] + B_0^{-1} u_t \qquad \ldots \qquad \ldots \qquad (3.2)$$

where '*g*' denotes the lag structure of predetermined variables.

(b) The Final Form

Since the reduced form includes lagged values of endogenous variables other than the one under consideration, we need to eliminate these by successive substitution. The final form resulting in this case expresses each endogenous variable in terms of an infinite distributed lag of exogenous variables.[5] This is

$$Y_t = -B(L)^{-1}\,C(L)\,Z_t + B(L)^{-1}\,u_t \qquad \ldots \qquad \ldots \qquad (3.3)$$

Finally, to determine the influence of exogenous shocks on the time path of endogenous variable (Y_t), it is desirable that all lagged values of *Y* should also be eliminated from the final form. Successive subtitution starting from the first period yields the final equations as follows:[6]

$$Y_t = \frac{b(L)\,C(L)}{B(L)}\,Z_t + \frac{b(L)}{B(L)}\,u_t \qquad \ldots \qquad (3.4)$$

[5]See Theil and Boot (1962) for details about the final form of econometric equation system.
[6]The final equations of the type (3.5) were suggested by Tinbergen (1939) and Goldberger (1959).

where $b(L)$ is the adjoint matrix of $B(L)$.[7]

(c) Conditions for Dynamic Stability

Before analysing the impact of policy variables on aggregate income, it is essential to prove that the model under consideration is dynamically stable. Model stability could be assessed by setting the right-hand side of the fundamental dynamic equation equal to zero. This manipulation yields the following characteristic equation.

$$B_0 Y_t^g + B_1 Y_t^{g-1} + \cdots + B_{g-1} Y_t + B_g = 0 \qquad \ldots \quad \ldots \quad (3.5)$$

The model will be dynamically stable if all roots of the characteristic equation have modules less than one.[8]

(d) Impact and Dynamic Multipliers

With no loss of generality the model (3.1) can be presented as a first order system of stochastic difference equations. That is:

$$A\, Y_t = BY_{t-1} + C Z_t + u_t \qquad \ldots \quad \ldots \quad \ldots \quad (3.6)$$

where

Y_t is a $m \times 1$ vector of dependent variables;
Z_t is a $k \times 1$ vector of exogenous variables;
B and C are the matrices of parameters with conformable dimensions; and
u_t is $m \times 1$ vector of error terms which are assumed to be serially uncorrelated.

This reduced form in this case will be

$$Y_T = \pi_1 Y_{t-1} + \pi_2 Z_t \qquad \ldots \quad \ldots \quad \ldots \quad (3.7)$$

where the Y_{t-1} is the lagged endogenous variable, Z_t is the set of pure exogenous variables, $\pi_1 = A^{-1}B$ is $m \times m$ matrix of coefficients for predetermined variables, and $\pi_2 = A^{-1}C$ is $m \times k$ matrix of coefficients for exogenous variables. Expression (3.7) can now be used to evaluate the change in aggregate income in response to a change in any of the exogenous variables. π_2 in this case will be the impact multipliers. Since a one shot or sustained shock in the exogenous variable continues to have a long-run effect on the dependent variables, the exact value of change could be

[7]See also Wallis (1973).

[8]The four possibilities of the solution can be found in Chiang (1984) and Pindyck and Rubinfeld (1981).

measured by computing multipliers for subsequent periods generally known as the dynamic multipliers. As stated by Wallis (1973), the values of the dynamic multipliers could be generated as follows:

$(I + \pi_1)\,\pi_2$ for the effect after one period

$(I + \pi_1 + \pi_1^2\;)\,\pi_2$ for the effect after two periods

. .

. .

. .

$(I + \pi_1 + \dots \pi_1^j)\,\pi_2$ for the effect after j periods.

If the matrix π_1 is such that $\pi_1^j \rightarrow 0$ as $j \rightarrow \infty$ then the total long-run or equilibrium multipliers will be:

$$\sum_{J=0}^{\infty} \pi_1^j \pi_2 = (\mathrm{I} - \pi_1)^{-1} \pi_2$$

where $(m, k)th$ element describes the change in equilibrium or long-run level of Y_m due to a unit change in Z_k.

IV. RESULTS OF ESTIMATION

To determine the consistency of the estimated parameters, the behavioural equations of the model have been estimated through different estimation techniques such as Ordinary Least Squares (OLS), Two Stage Least Squares (2SLS), Three Stage Least Squares (TSLS) and Multivariate Estimation (ME). For fast convergence in TSLS and ME, the estimated values of parameters obtained from 2SLS have been used as "initial guesses".

(a) Regression Results

The results of estimation pertaining to the behavioural equations are presented in Table 1. Various estimates for the consumption function reveal that the short-run mpc out of disposable income (net of remittances) ranges between 0.32 to 0.56 and the short-run mpc out of remittances ranges between 0.12 and 0.41. On the other hand, the long-run mpc out of disposable income ranges between 0.81 and 0.89 and the same out of remittances ranges between 0.19 and 0.89. On the basis of summary statistics the results show fairly consistent behaviour across alternative estimation techniques.

The estimates of the tax function indicate that the marginal revenue contribution of direct taxes with respect to non-agricultural income (tax base) is only 0.023 or less. These taxes are essentially inelastic to the base as the elasticity estimate range below unity under alternative estimation techniques.[9]

[9]Similar results for income and corporate taxes were found in Naqvi *et al.* (1983) and Naqvi, Khan and Ahmed (1992).

Table 1

Estimation of Behavioural Equations of the Model

	OLS	2SLS	TSLS	ME
ESTIMATES OF CONSUMPTION FUNCTION				
Intercept	188.23	318.5	–12.92	–1052.10
	(0.35)	(0.58)	(–0.03)	(451.59)
Y^{dn}	0.50	0.382	0.321	0.5
	(4.37)	(2.86)	(3.44)	(7.47)
R^m	0.41	0.391	0.336	0.116
	(2.6)	(2.44)	(2.94)	(1.13)
C_{t-1}	0.384	0.532	0.624	0.375
	(2.61)	(3.12)	(5.64)	(4.02)
$\bar{R}^2$	0.99	0.99	0.998	0.84
D.W.	1.40	1.53	1.545	0.61
SER	778.02	794.96	754.30	228.06
ESTIMATES OF TAX FUNCTION				
Intercept	126.05	102.81	83.12	88.82
	(0.88)	(0.86)	(1.09)	(1.21)
$y$na	0.022	0.023	0.023	0.019
	(6.37)	(7.49)	(11.74)	(10.36)
$\bar{R}^2$	0.60	0.63	0.85	0.85
D.W.	2.06	2.02	0.96	0.61
SER	163.37	160.53	182.61	228.501
ESTIMATES OF INVESTMENT FUNCTION				
Intercept	458.28	501.94	163.38	367.48
	(1.92)	(1.95)	(0.68)	(1.72)
ΔY	0.142	0.158	0.076	0.127
	(2.73)	(2.48)	(1.50)	(3.44)
r	–50.917	–58.42	–21.16	–34.47
	(–1.77)	(–1.76)	(–0.84)	(–1.73)

Continued–

Table 1 – (*Continued*)

	OLS	2SLS	TSLS	ME
$\bar{R}^2$	0.898	0.898	0.91	0.92
D.W.	1.86	1.89	1.74	1.47
SER	308.80	309.61	286.2	300.45
ESTIMATES OF IMPORT FUNCTION				
Intercept	198.65	198.68	210.65	185.39
	(0.88)	(0.88)	(0.94)	(0.85)
Y	0.006	0.067	0.071	0.08
	(3.13)	(3.16)	(4.08)	(5.21)
M_{t-1}	0.44	0.43	0.39	0.362
	(2.34)	(2.25)	(2.58)	(2.67)
$\bar{R}^2$	0.97	0.97	0.97	0.96
D.W.	1.82	1.80	1.74	1.31
SER	505.94	505.97	487.03	554.99
ESTIMATES OF MONEY DEMAND FUNCTION				
Intercept	–85.77	–387.53	–199.08	279.5
	(–0.2)	(–0.53)	(–0.28)	(0.45)
Y	0.210	0.22	0.19	0.197
	(3.45)	(3.58)	(3.48)	(3.97)
r	–93.71	–40.31	–36.78	–141.25
	(–0.93)	(–0.36)	(–0.36)	(–1.711)
m^{d}_{t-1}	0.49	0.44	0.52	0.57
	(2.82)	(2.49)	(3.27)	(4.1)
$\bar{R}^2$	0.98	0.98	0.99	0.99
D.W.	1.68	1.68	1.81	1.1
SER	1038.02	1044.93	979.44	1240.14

Note: t-values are reported in the parentheses.

The investment function includes an accelerator term which is statistically significant in all cases. The estimated coefficients suggest that a one unit increase in change in income leads to an increase in private investment within the range of 0.08 and 0.16 units depending upon the choice of estimation technique. The results concerning the rate of interest also confirm the Keynesian view. The highly aggregative imports function reveals that import demand is quite sensitive to changes in income. While the short-run marginal propensity to import (*mpi*) ranges between 0.06 and 0.08, the long-run *mpi* remains close to 0.12. However, balance of payments problems could arise if the import bill continues to grow with the increase in income over time. Finally, the estimates for money demand show that the demand increases proportionally with an increase in income and the reverse is true for the interest rate. Both these results are theoretically correct. The lag-dependent variable ranges between 0.44 and 0.57 and in all four cases this variable is statistically significant.

(b) Dynamic Analysis

In the present paper, we have adopted Wallis's procedure (1973) to establish the stability of the model and to derive the impact and the dynamic multipliers. In this respect, stability of the model is determined by calculating the eigen values of π_1 in (3.7). Supposing that the eigen values are given by λ_i where $i = 1, 2, \ldots m$ (where $m^1 < m$), then stability requires that all non-zero eigen values of $det\ [(B_0)^{-1} B - \lambda I)] = 0$ be less than one in modules, with I being an $m \times m$ identity matrix.[10]

For the model specified above, we obtained four non-zero latent roots of which two are real and the remaining two are complex conjugates. That is

$$\lambda_1 = 0.394$$

$$\lambda_2 = -0.00013$$

$$\lambda_3, \lambda_4 = 0.723 \pm 0.04\, i$$

Since the magnitude of these roots (including λ_3 and λ_4) is less than unity, the solution will be *stable* but oscillatory.

The dynamic multipliers for the time path of GNP are presented in Table 2. These multipliers which converge to zero with the increase in the length of time lags also exhibit damped oscillations. Furthermore, positive changes in government expenditure and money supply have a stimulating effect on GNP although the impact effect of the former is greater than the latter. It may be noted, however, that the effect of the money supply operates through the interest rate in the investment demand function. The long-run multipliers for government expenditure and money

[10]This procedure in similar to one discuss in Section III.

supply are obtained by summing the impact and the dynamic multipliers in Table 2. The long-run multipliers for these two policy variables are 1.9298 and 1.1255 respectively. These results confirm that both government expenditure and money supply are effective policy tools is influencing the target variables. Although the superiority of either of these instruments from such a simple model will be a tall claim, nonetheless, the results from a largely Keynesian-type model favours fiscal interventions.

Table 2

Dynamic Multipliers for the Time Path of GNP

No. of Lags	Multipliers of	
	Government Expenditure	Money Supply
0	2.1985	0.5364
1	0.5482	0.2685
2	0.2103	0.1953
3	–0.0106	0.1283
4	–0.1361	0.0731
5	–0.1906	0.0324
6	–0.1975	0.0048
7	–0.1762	–0.0116
8	–0.1414	–0.0196
9	–0.1033	–0.0200
10	–0.0682	–0.0201
11	–0.0393	–0.0166
12	–0.0177	–0.0125
13	–0.0030	–0.0085
14	0.0050	–0.0051
15	0.0101	–0.0026
16	0.0113	–0.0006
17	0.0105	0.0004
18	0.0087	0.0010
19	0.0066	0.0012
20	0.0045	0.0013
	Σ 1.9298	Σ 1.1255

V. SUMMARY AND CONCLUSIONS

A simple linear yet dynamic model is specified in this paper to lay down the conditions of dynamic stability and to evaluate the relative importance of fiscal and monetary policy instruments. The model stability was determined by characteristic roots which all had modules less than unity. The stability of the model was further confirmed by the values of dynamic multipliers which converged to zero as the length of the time lag increased. It was observed that both government expenditure and money supply were effective tools in stimulating GNP although the effect of these policy tools became mild as the time lag increased.

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